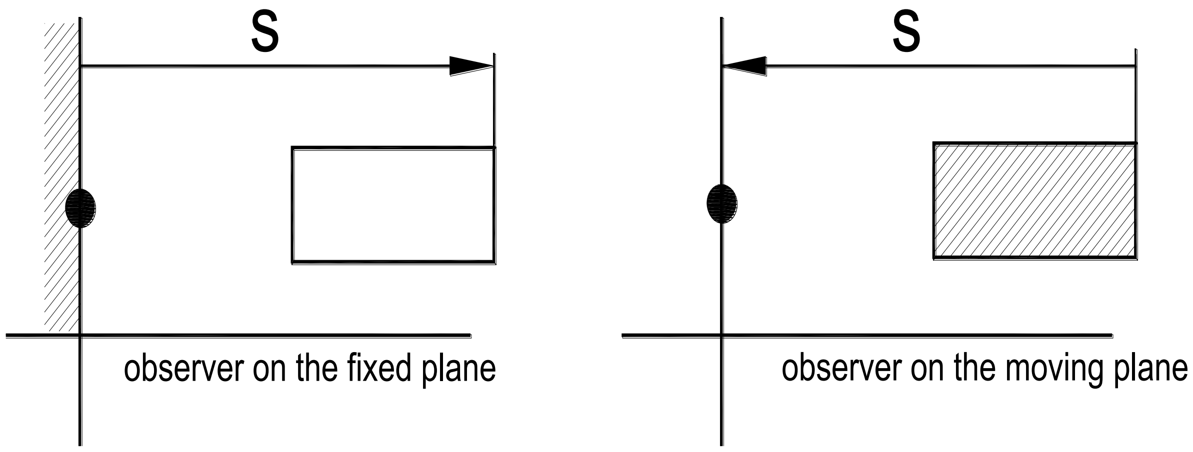
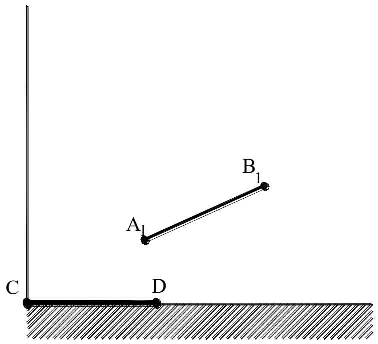
INVERSE MOTION and RELATIVE MOTION

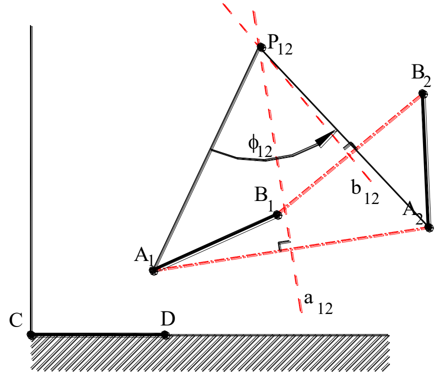
Inverse Motion

Change the point of observation

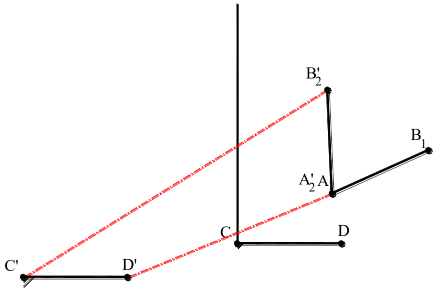


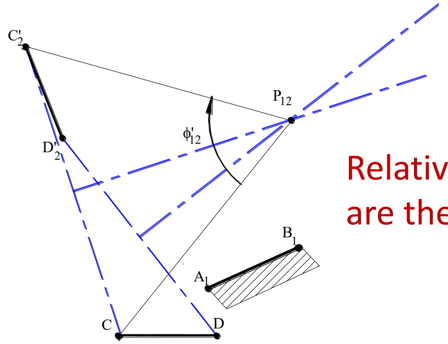
Two positions of a moving plane





The motion of AB relative to CD for 2 positions



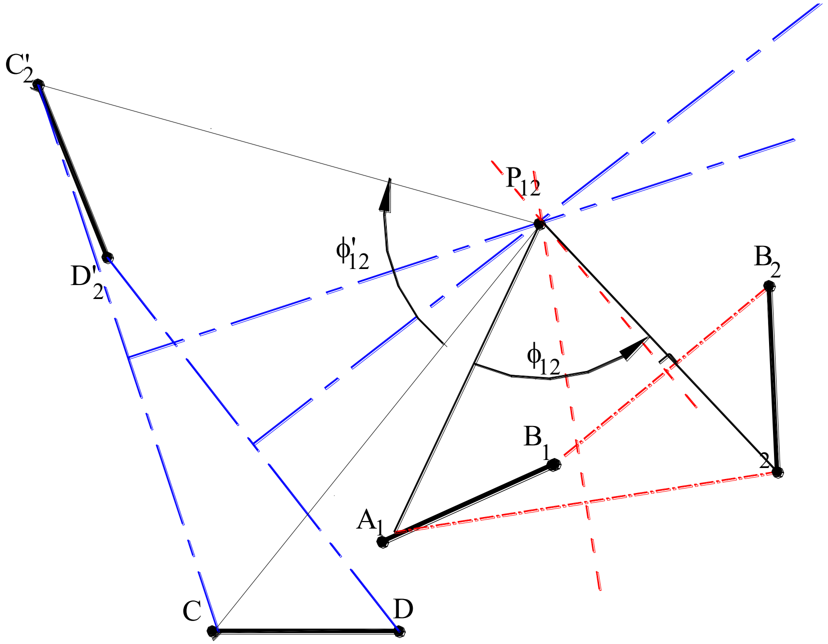


Relative positions are the same

To invert the motion, fix CDA_2B_2 to each other.

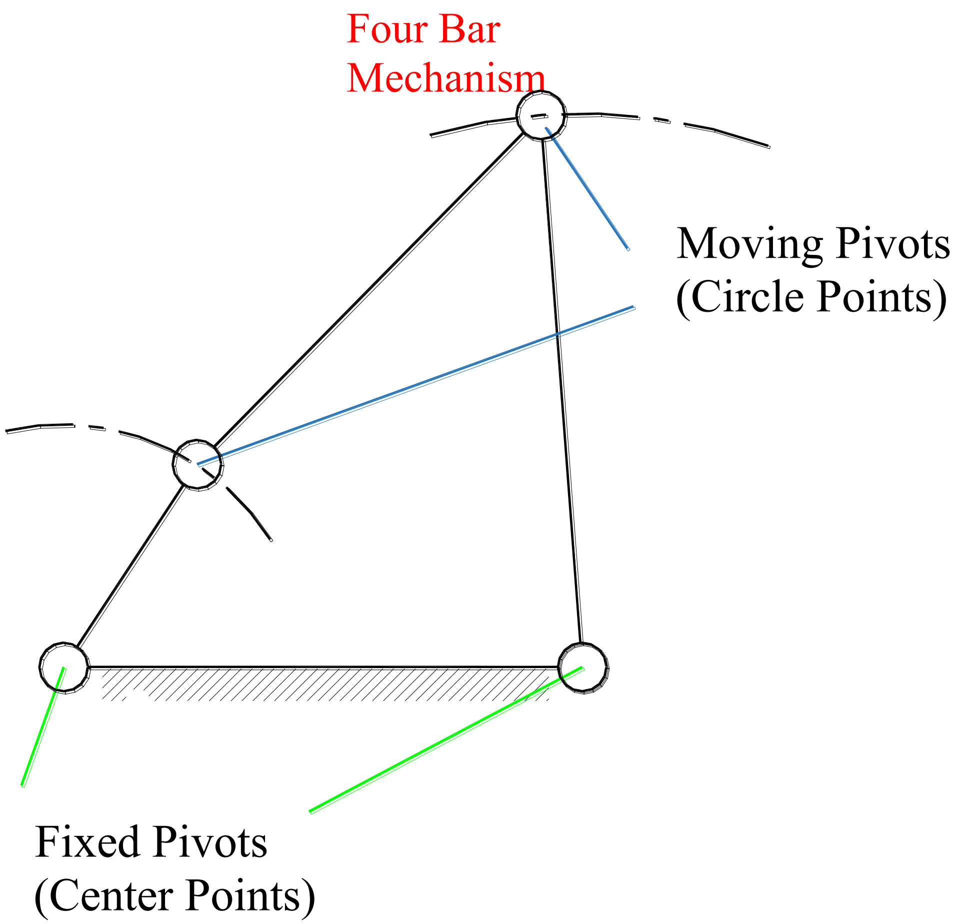
1. Translate so that A_2’ coencides with A_1. (CD will move to C’D’).
2. Rotate so that B'_2 coencides with B_1 (CD is at $C'_2D'_2$.

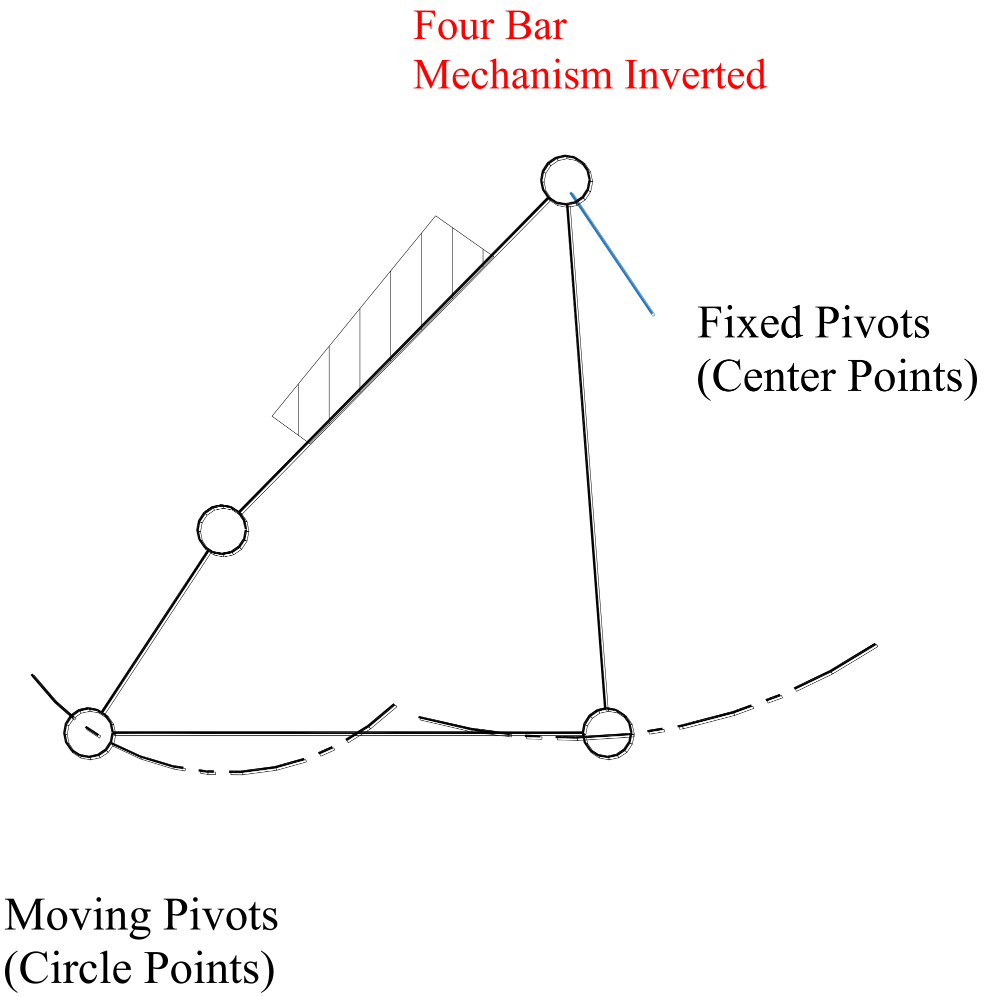
Motion from CD to $C'_2D'_2$ is the inverse motion of the plane CD relative to AB



For the inverse motion:

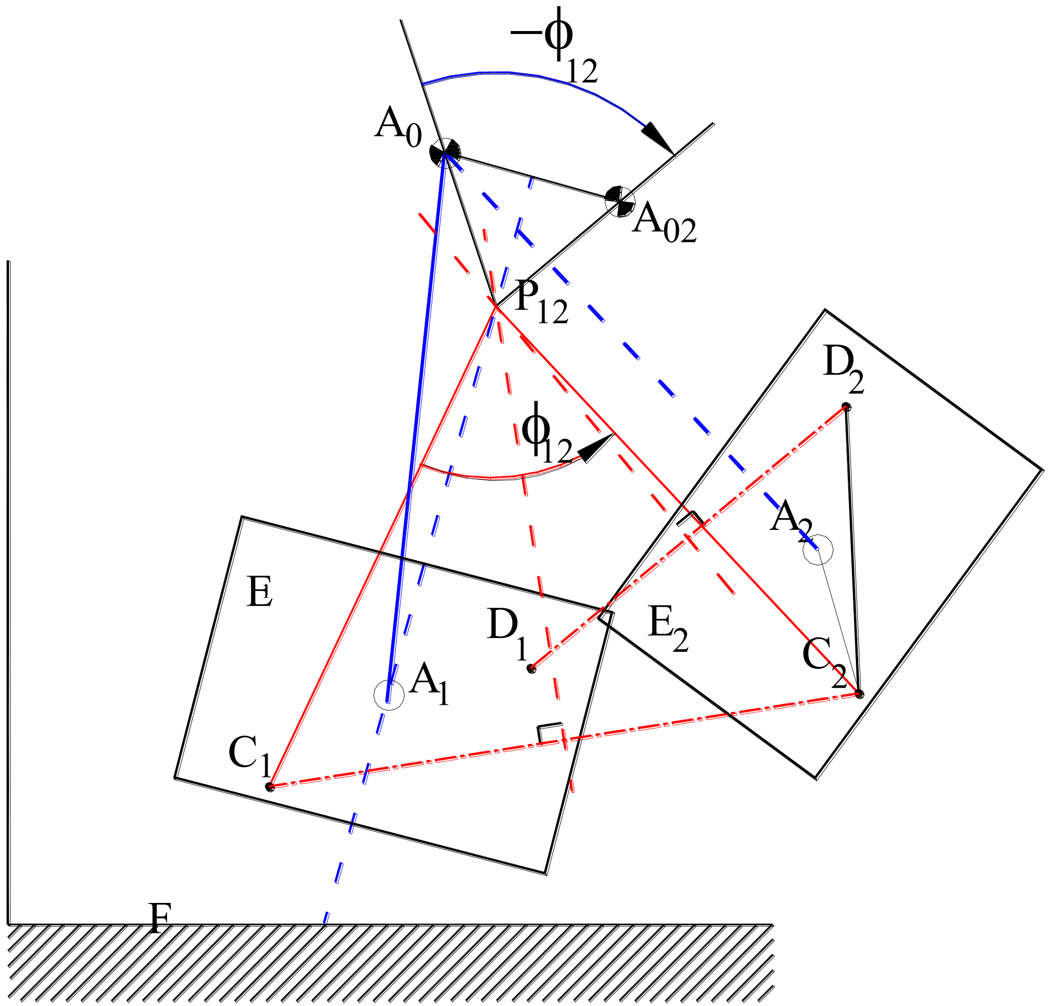
1. The pole P_{12} is the same as the original motion
2. The angular rotation is equal to the original motion but in reverse direction: $\phi'_{12}=-\phi_{12}$





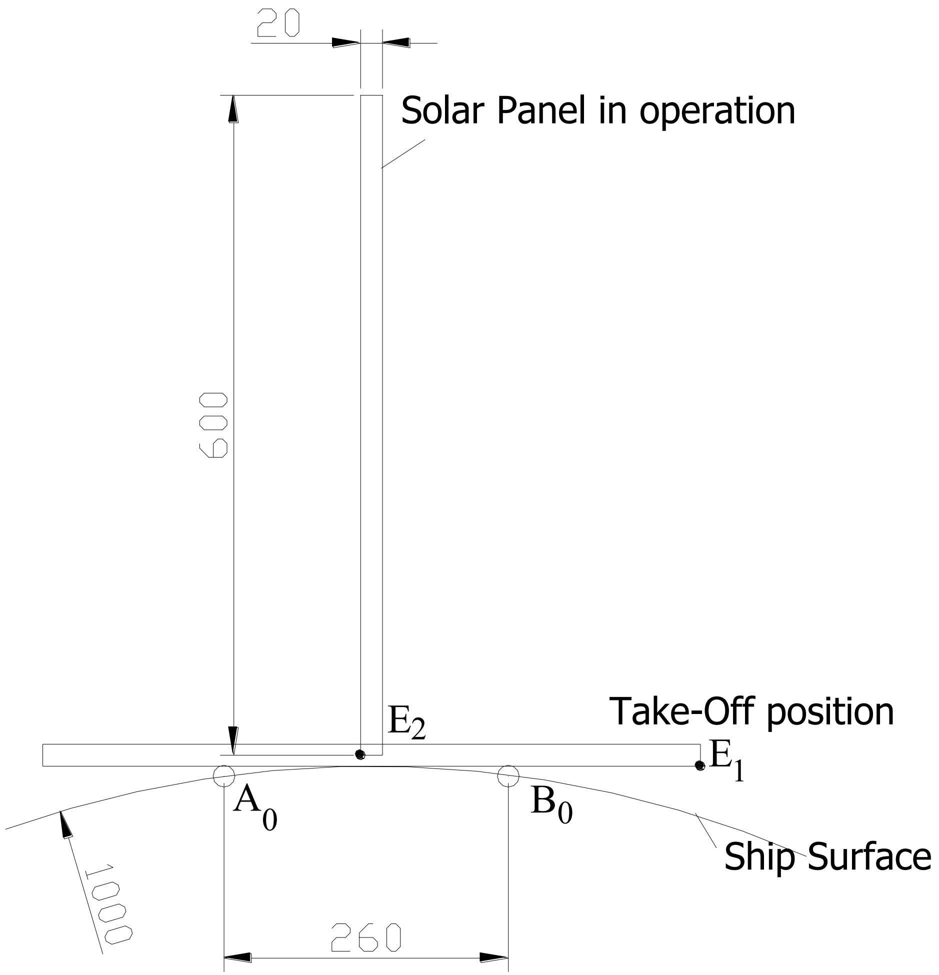
In Kinematic inversion Circle and Center points change their role!!!

Using kinematic inversion you can first select center point A_0 and then determine circle point A_1



Geogebra

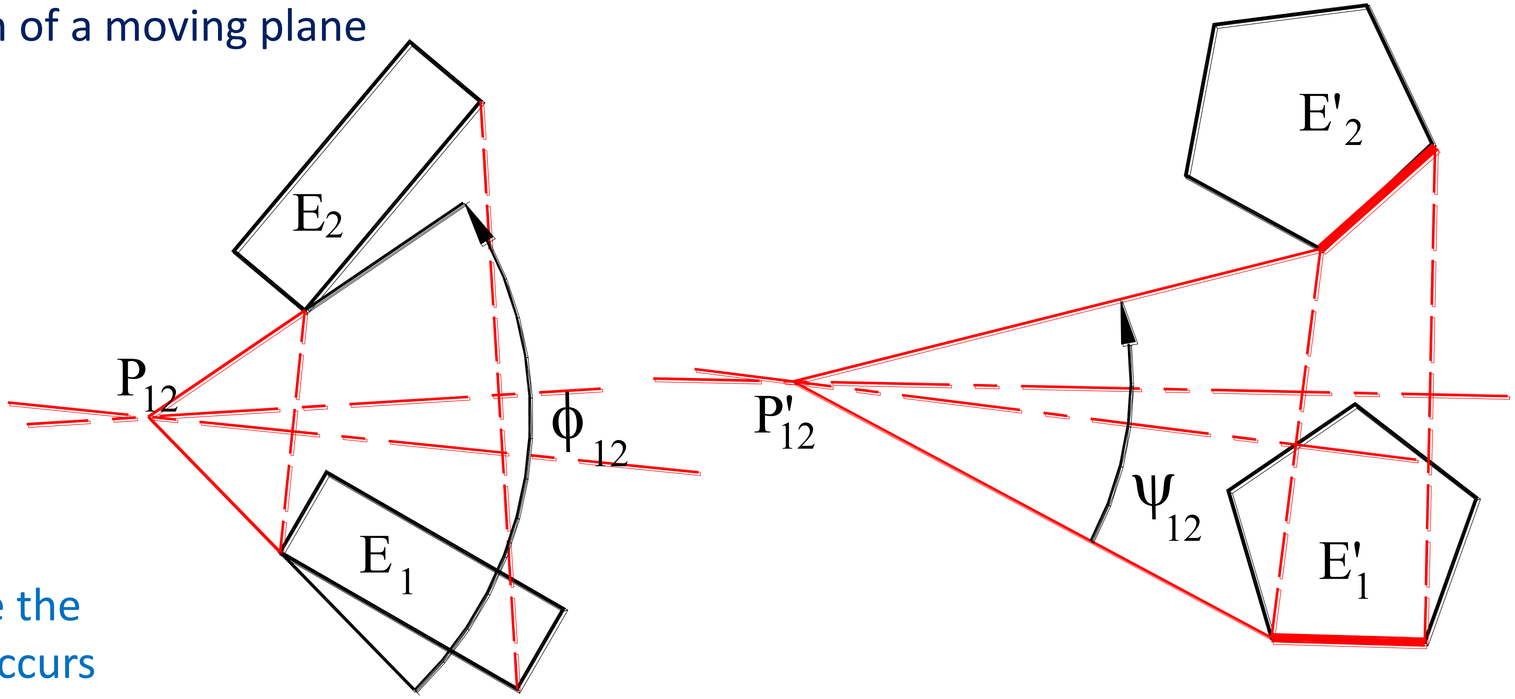
You are to have solar panels for a space ship. During takeoff, the panels must be tangent and when the spaceship is in orbit, the panels must be perpendicular to the space ship. The fixed pivots must be on the surface of the capsule as shown. Design a four-bar mechanism to move the solar panels in between two positions.



Relative Motion

Relative Motion between two moving planes.
You are to observe the motion of a moving plane from another moving frame.

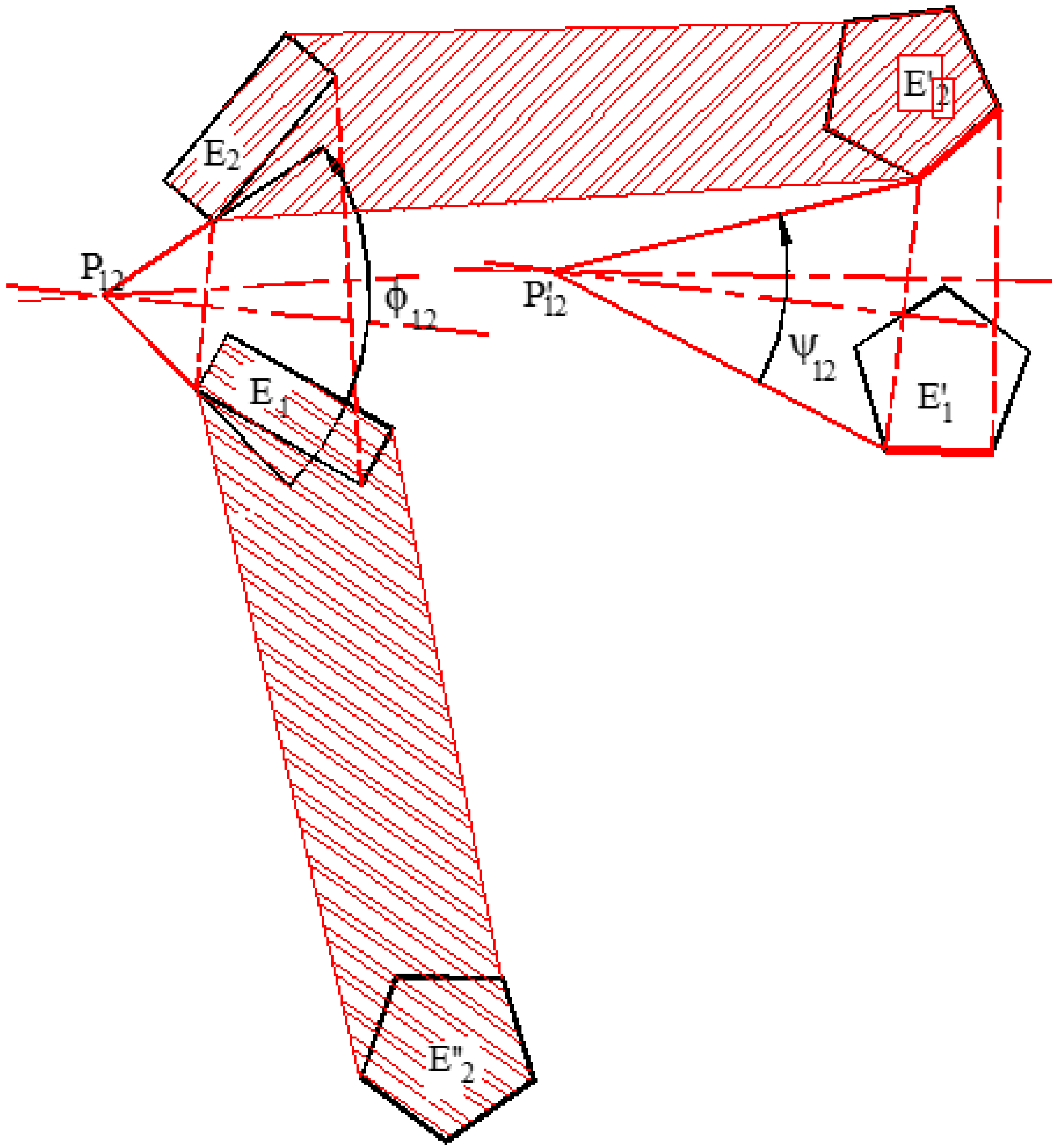
At time $t=t_1$, planes are at E_1 and E'_1. At $t=t_2$ planes moved to E_2, E'_2. How would you see the motion if you were sitting on plane E?



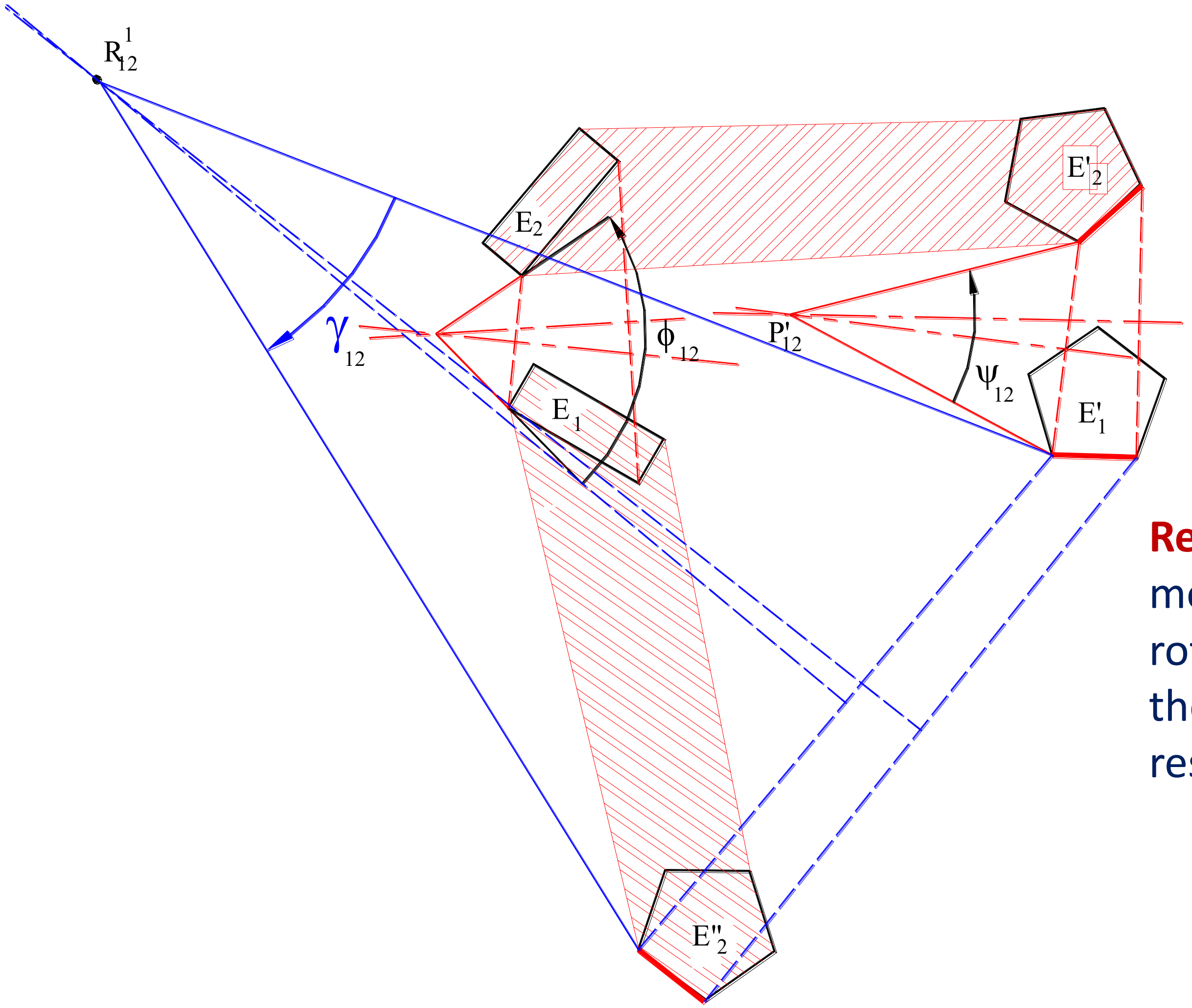
If you were sitting on the fixed frame the motion of two planes most simply occurs by rotation about poles P_{12} and P'_{12} by angles ϕ_{12} and ψ_{12}.

To see the motion of E’ relative to E, ‘’SUBTRACT’’ the motion of E from that of E’.

To ‘’subtract’’ the motion, Fix E_2 and E'_2 to each other and rotate both planes about P_{12} by an angle $-\phi_{12}$. This moves E’ to E_1 and E'_2 to E''_2. The motion from E'_1 to E''_2 is what you will see if you were sitting on plane E.



There are two positions of E’ relative to E: E’1 and E’’2. Determine the pole for this relative motion and let us call this pole R^1_{12} , since it is a ‘’relative pole’’



$$\gamma_{12} = \psi_{12} - \phi_{12}$$

Relative Motion: When E is at E_1, the motion of the plane E’ from E'_1 to E''_2 by a rotation about R^1_{12} by an angle $\psi_{12} - \phi_{12}$ is the relative motion of the plane E’ with respect to plane E when E is at E_1

The motion of the planes E and E' from E_1 to E_2 and from E'_1 to E'_2 can be realized by:

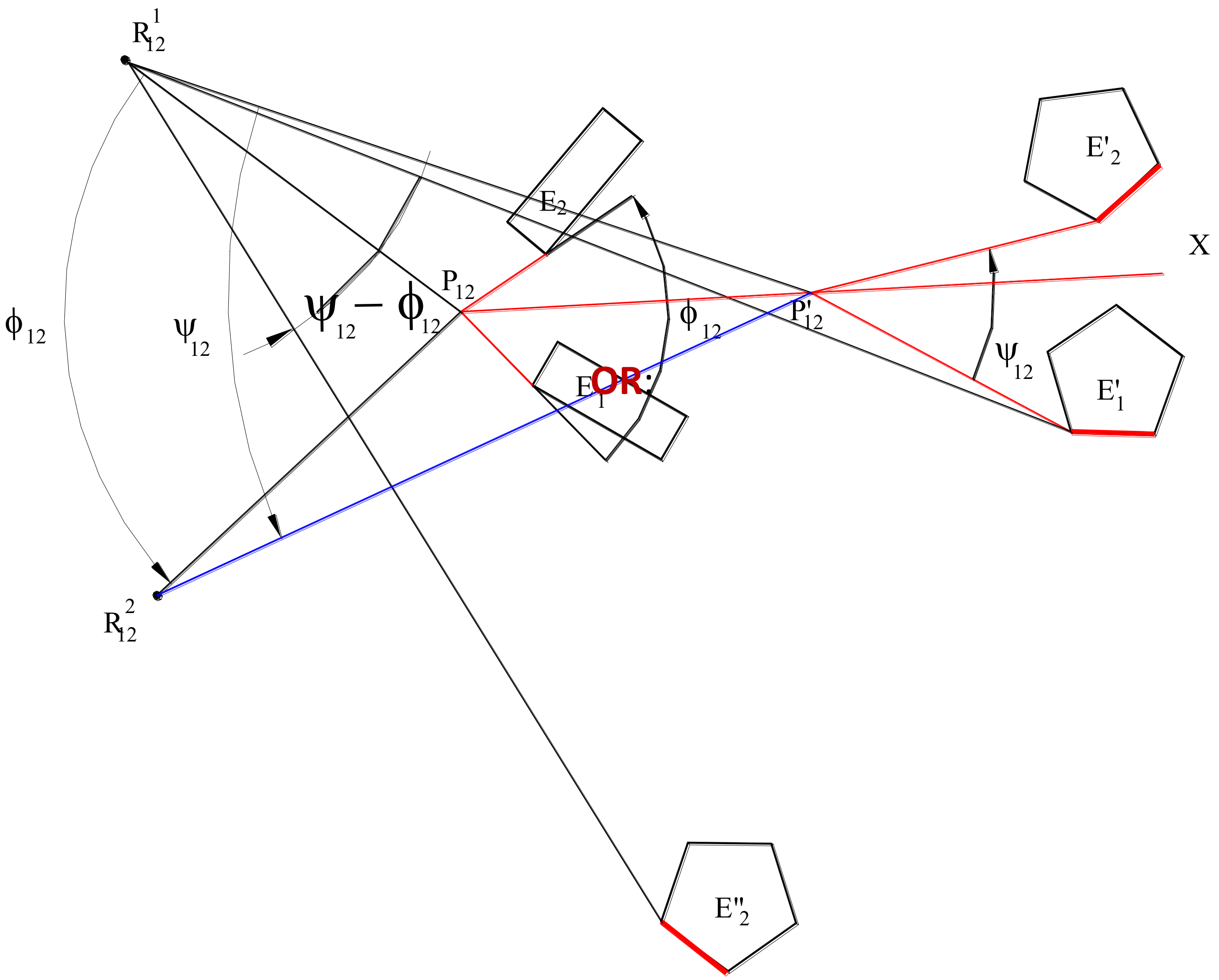
Rotate E about P_{12} by an angle ϕ_{12} and rotate E'_1 about P'_{12} by an angle ψ_{12}.

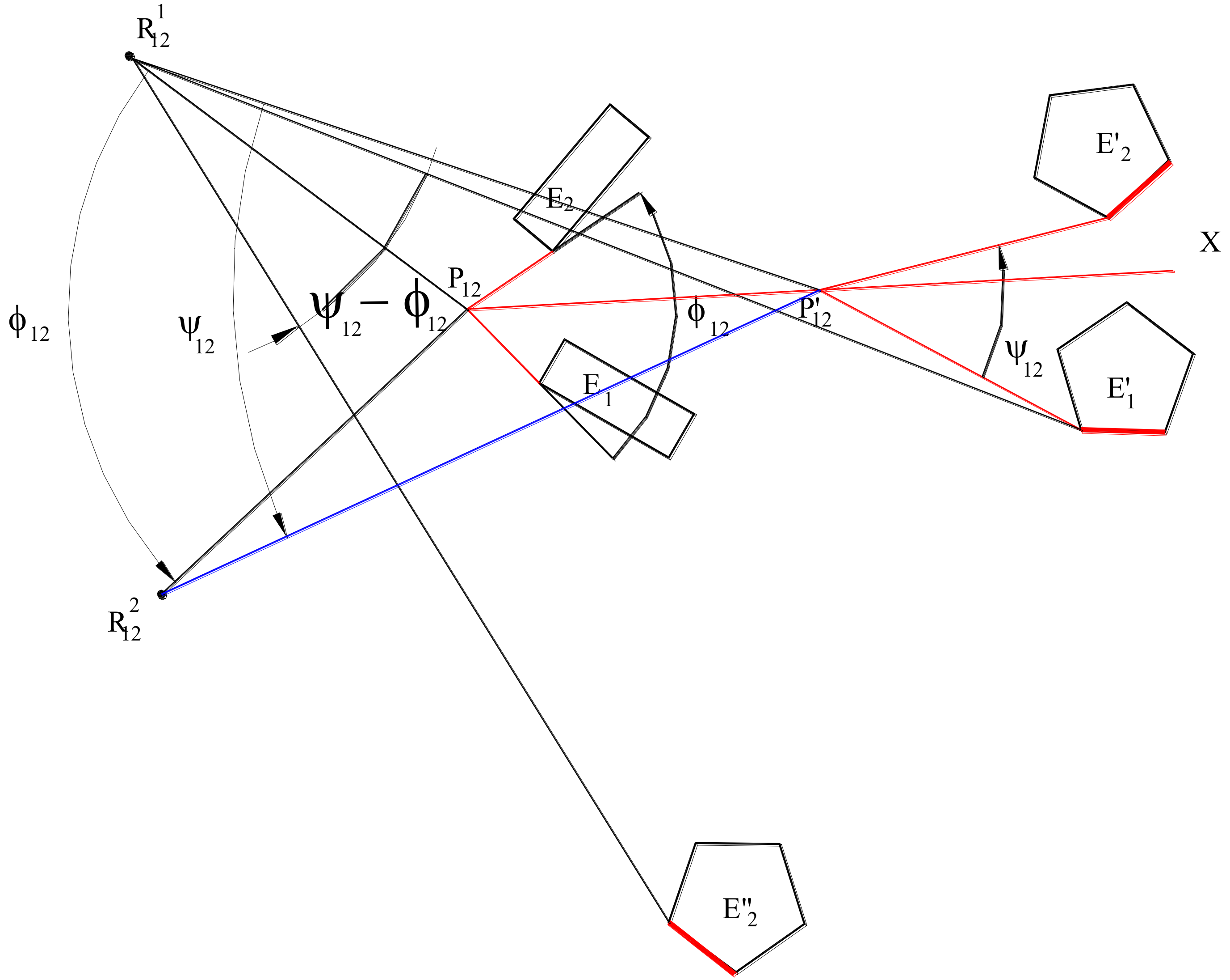
a) Rotate E'_1 about R^1_{12} by $\gamma_{12}=\psi_{12}-\phi_{12}$. (This motion moves E' from E'_1 to E''_2)
b) Fix E and E' and rotate about P_{12} by an angle ϕ_{12}.

The line $P_{12}R^1_{12}$ will also rotate by an angle ϕ_{12} about P_{12} and will be at R^2_{12} in the second position of the planes.

You can also move R^1_{12} to R^2_{12} by rotating $P'_{12}R^1_{12}$ about P'_{12} by ψ_{12}!!

R_{12} is not the same for the two positions!!





$P_{12}R^1_{12}=P_{12}R^2_{12}$
$P'_{12}R^1_{12}=P'_{12}R^2_{12}$

$P_{12}R^1_{12}P'_{12} = P_{12}R^2_{12}P'_{12}$ (SSS)

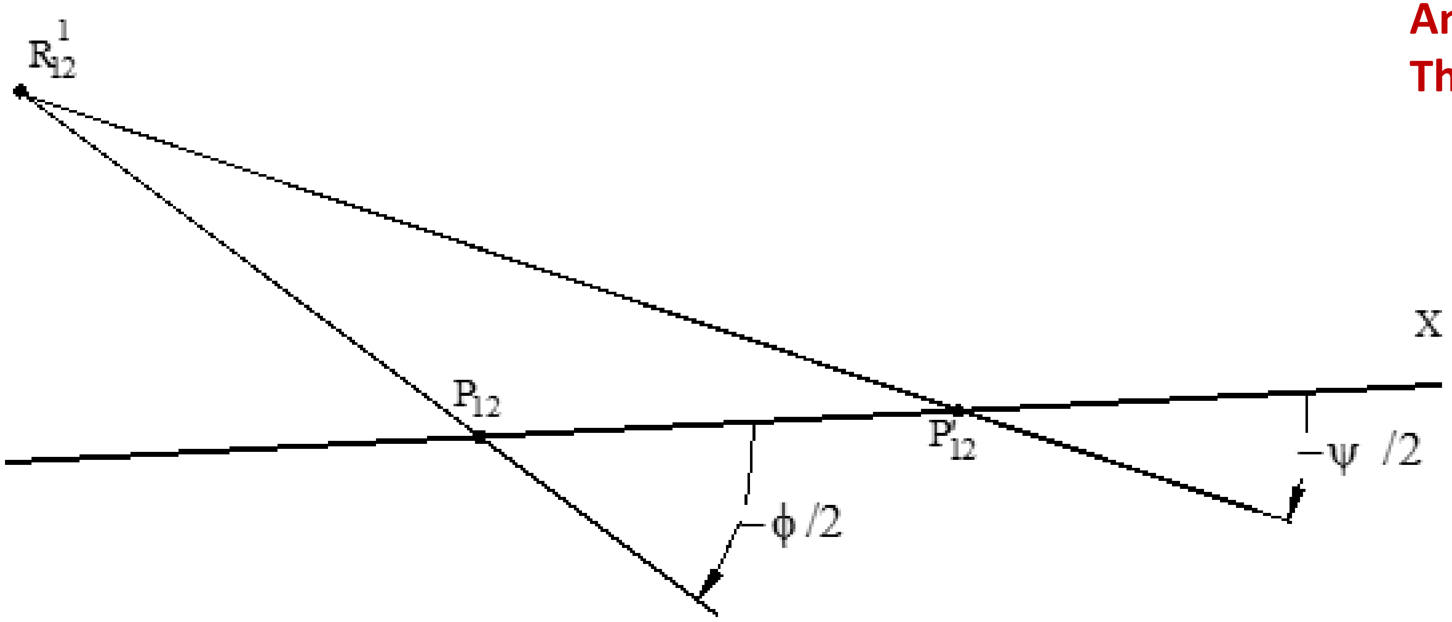
And $<R^1_{12}P_{12}R^2_{12}= \phi_{12}$
$<R^1_{12}P'_{12}R^2_{12}= \psi_{12}$

Hence

$<XP_{12}R^1_{12}= -\phi_{12}/2$
$<XP'_{12}R^1_{12}= -\psi_{12}/2$

X is a directed line segment in the direction of $P_{12}P'_{12}$.

To determine the relative pole :



Angles are directed angles.
Therefore their direction is important..

1. Determine the rotation poles of the planes E and E' and the angles of rotation between the two positions.
2. Draw a line making an angle $-\phi_{12}/2$ from P_{12} with respect to the line $P_{12}X$. Draw another line from P'_{12} that makes an angle $-\psi_{12}/2$ with respect to $P_{12}X$.
3. The intersection of the two lines drawn will locate the location of the relative pole in position 1, R^1_{12}, of the moving planes.

The location of the relative pole in second position: R^2_{12}

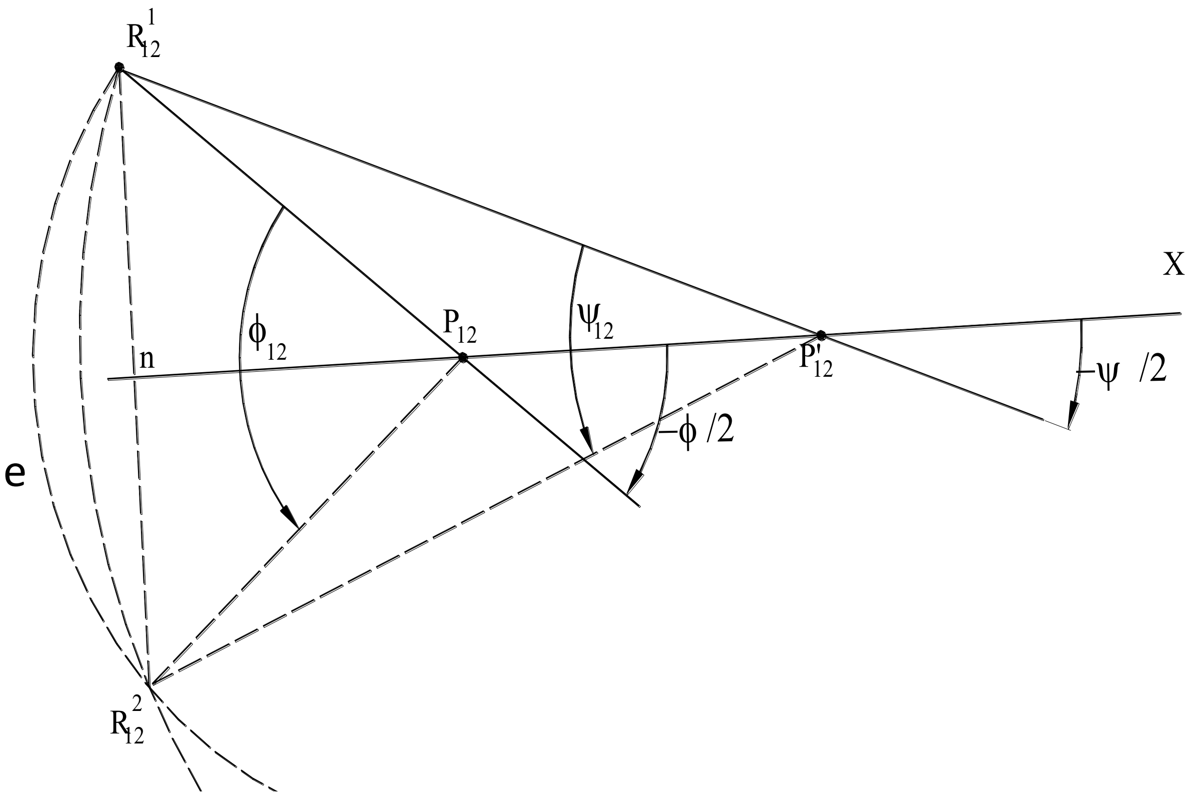
Relative pole is a common point of the two planes for the two positions..

Hence:

1. Rotate the line $P_{12}R^1_{12}$ about P_{12} by an angle ϕ_{12} .

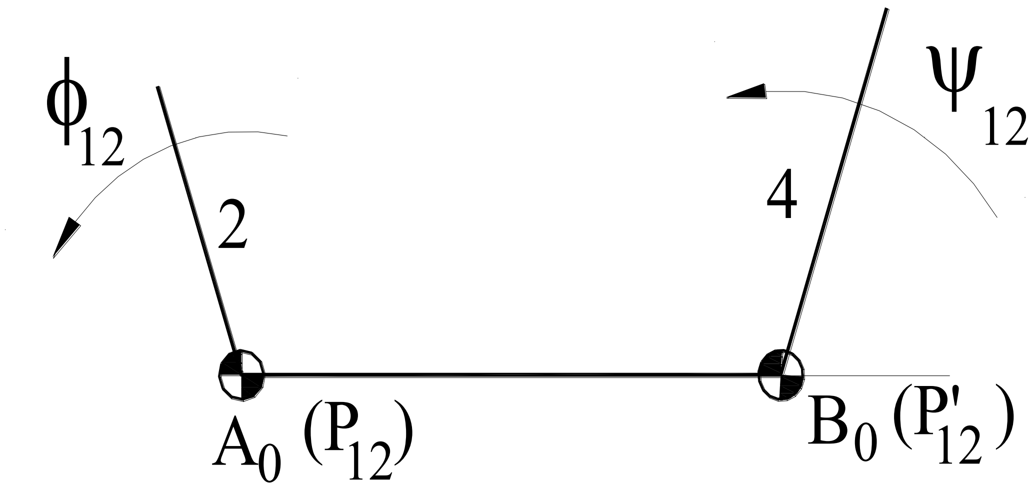
2. Rotate the line $P'_{12}R^1_{12}$ about P'_{12} by an angle ψ_{12}

3. Take the image of R^1_{12} about the line $P_{12}P'_{12}$ (Draw a line from R^1_{12} perpendicular to $P_{12}P'_{12}$ and select $R^1_{12}n=R^2_{12}n$.)



All the three methods will result in R_{12}^2 .

Correlation of Crank Angles



Given: the correlated rotation of two cranks: as one crank rotates by an angle ϕ_{12} the other crank has to rotate by an angle ψ_{12}

Find: A Four-bar mechanism to perform this task.

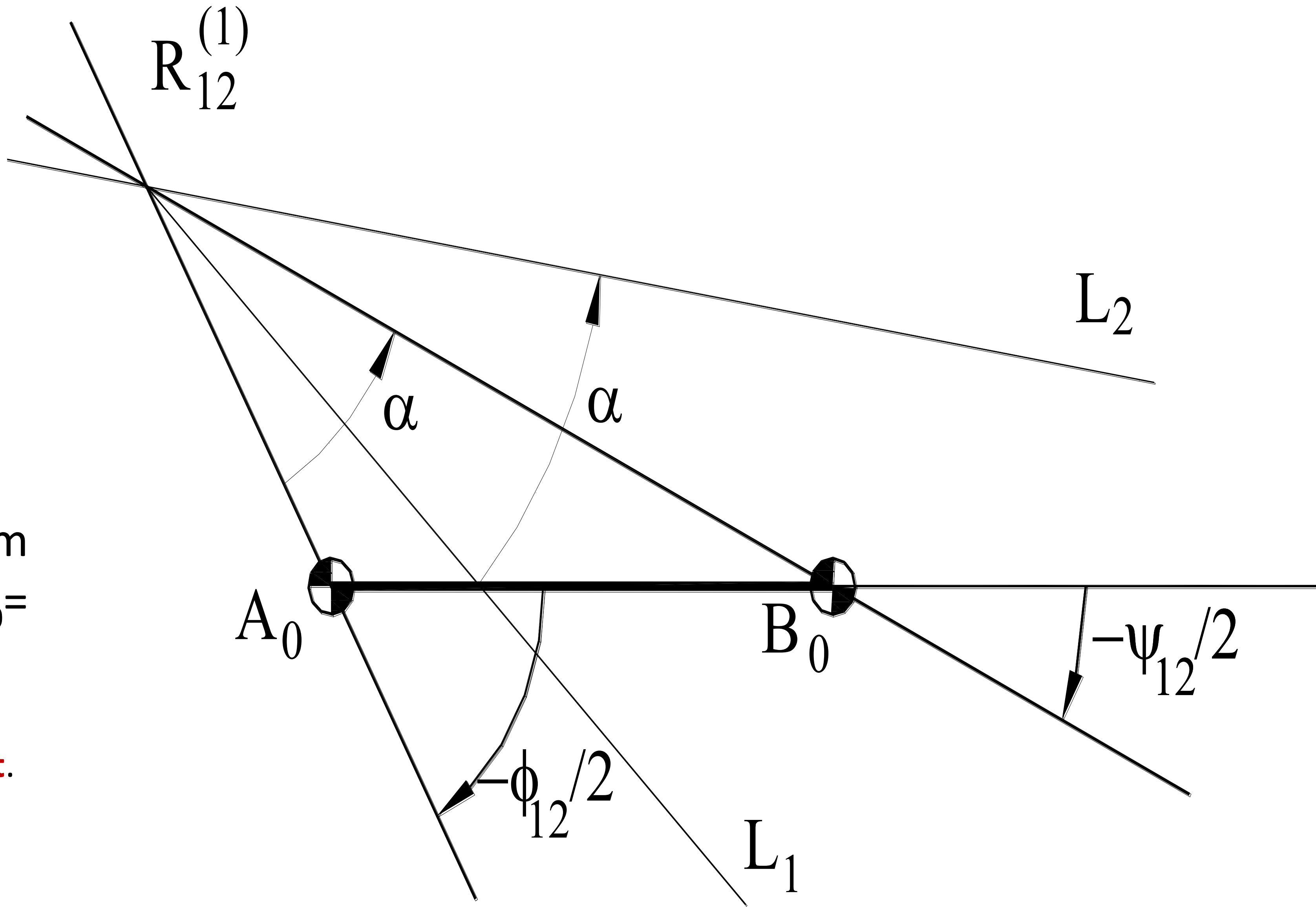
Method 1

1. Locate $R_{12}^{(1)}$

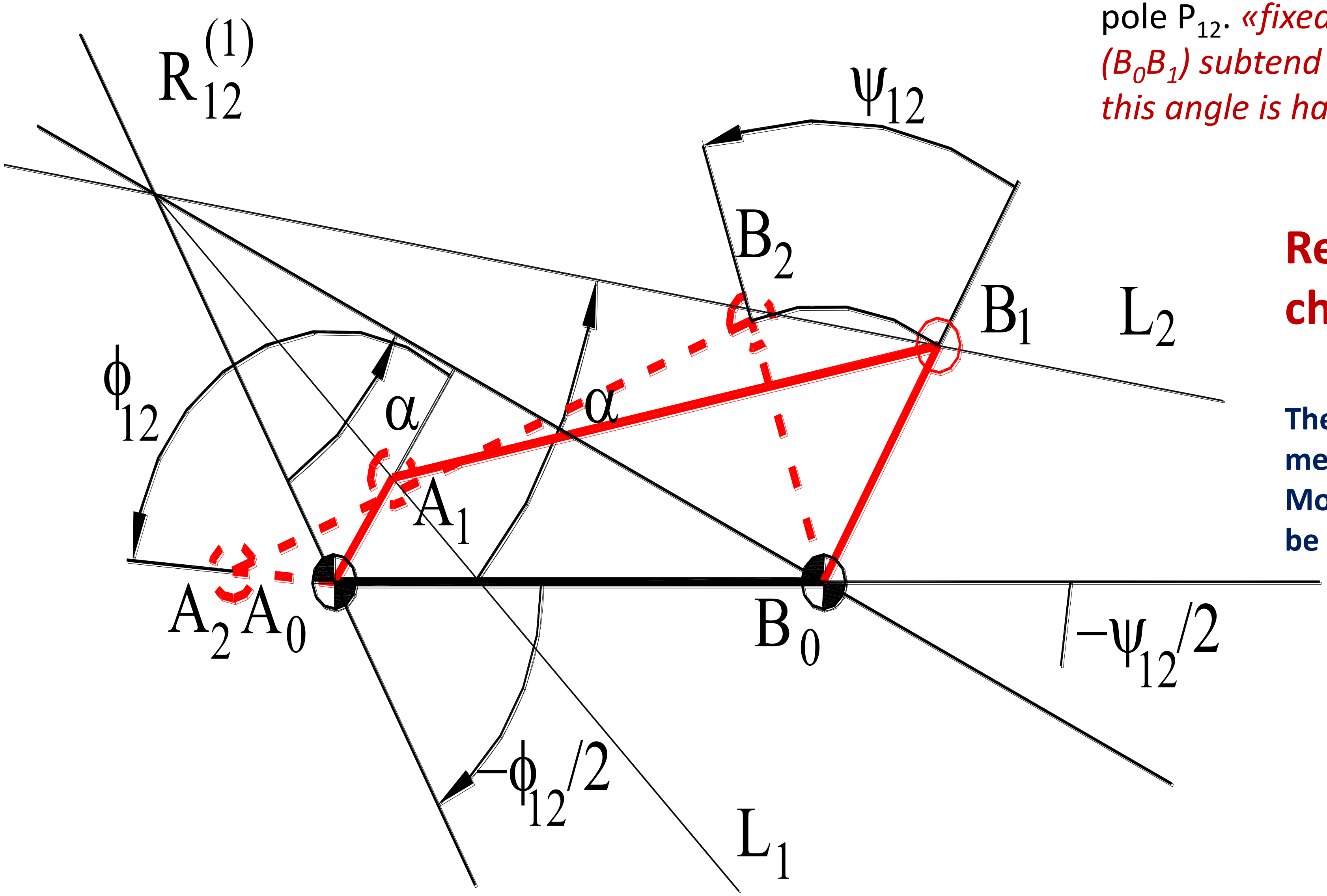
2. Draw an arbitrary line L_1 from $R_{12}^{(1)}$

3. Draw another line L_2 from $R_{12}^{(1)}$ such that $\angle A_0 R_{12}^{(1)} B_0 = \angle L_1 R_{12}^{(1)} L_2$

Direction of the angles are important.



4. Select A_1 anywhere on L_1 and select B_1 anywhere on L_2

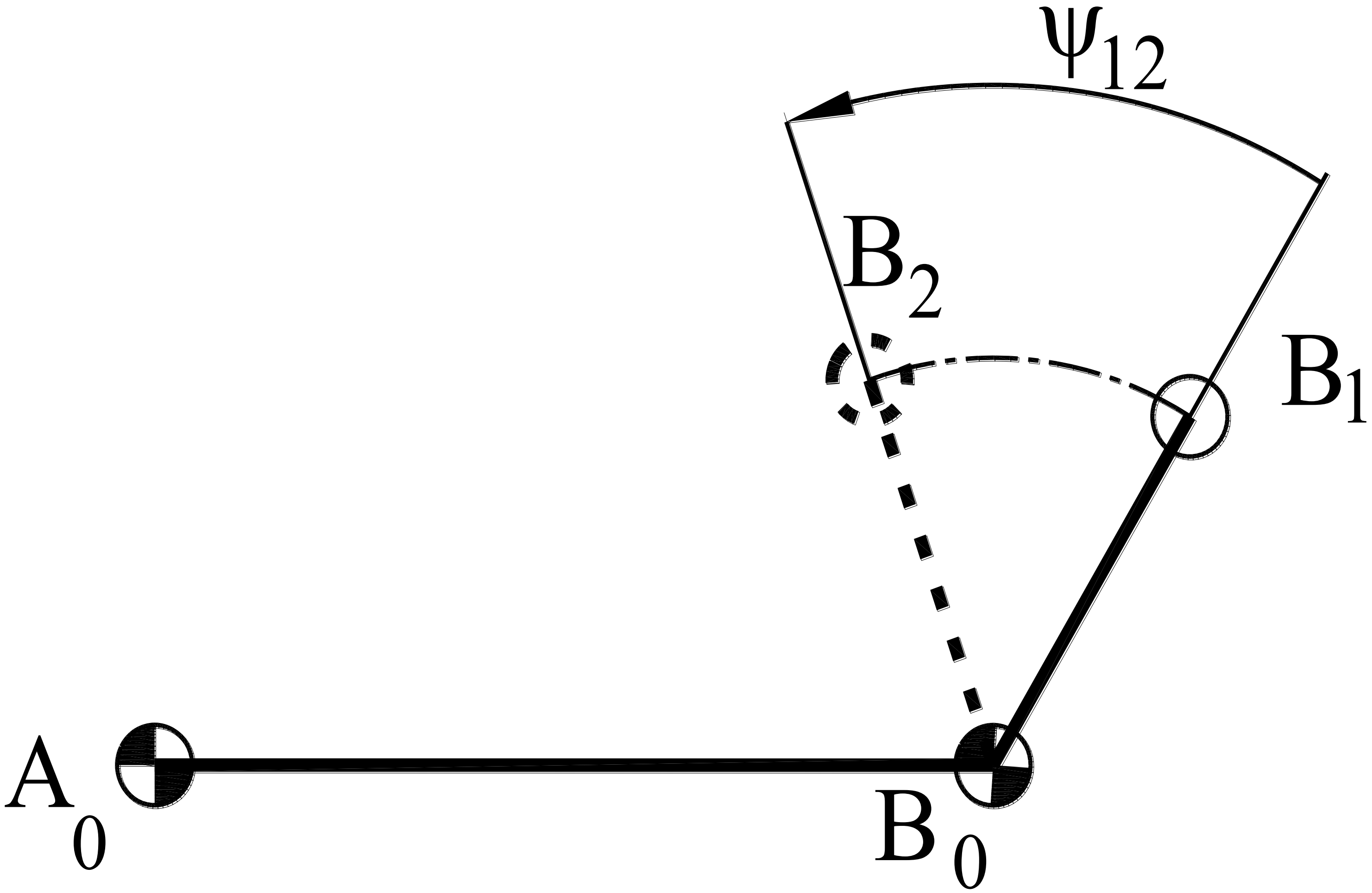


Relative motion is the motion of A0B1 relative to A_0A_1. If A_0A_1 was fixed, R^1_{12} is the pole P_{12}. *«fixed link (A_0A_1) and the Coupler (B_0B_1) subtend equal angles at the pole and this angle is half the rotation angle»*

Result Must always be checked!!!!!!

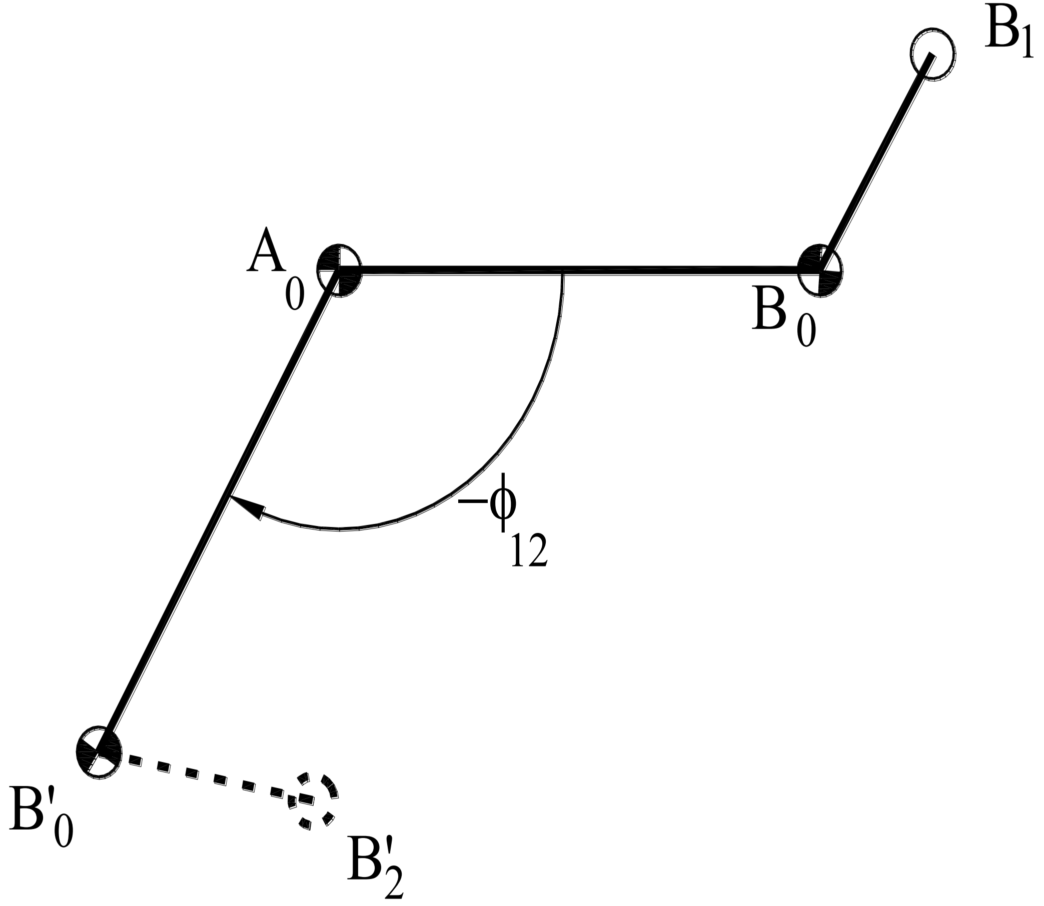
The method only tells you the mechanism exists for two positions. Motion in between the positions must be checked.

Method 2

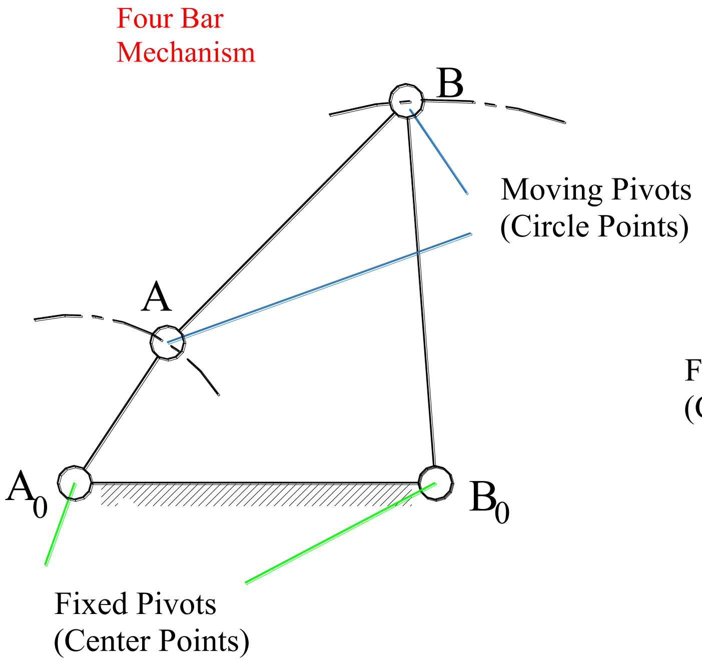


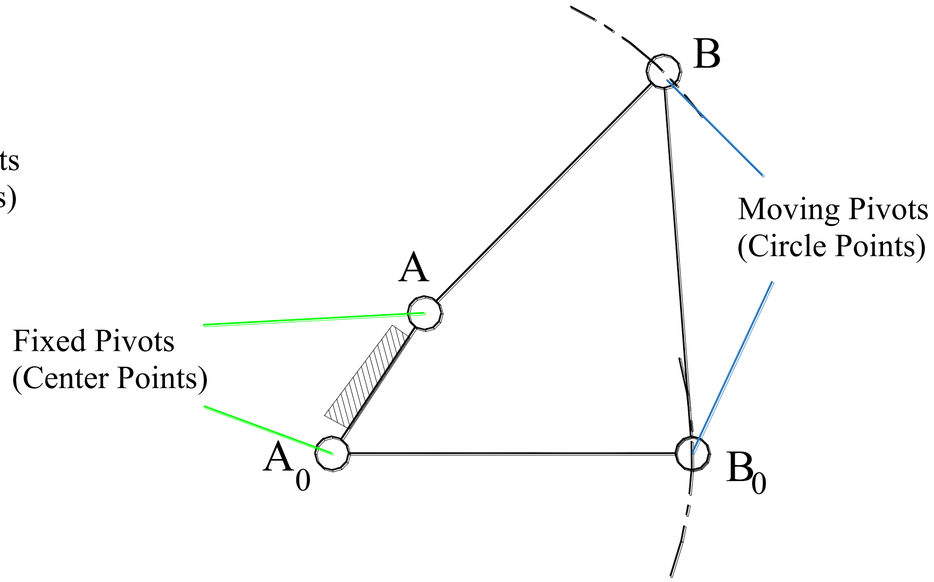
1. Select B_1 anywhere you like (if A_0B_0 is not given, you can arbitrarily select A_0B_0). Determine B_2 so that $\angle B_2B_0B_1 = \psi_{12}$ is satisfied.).

2. Fix A_0B_0 and B_0B_2 to each other and rotate about A_0 by an angle $-\phi_{12}$. We have «subtracted» the motion of A_0A from the motion of B_0B. The motion from B_0B_1 to $B_0B'_2$ is the motion of the plane B_0B relative A_0A.

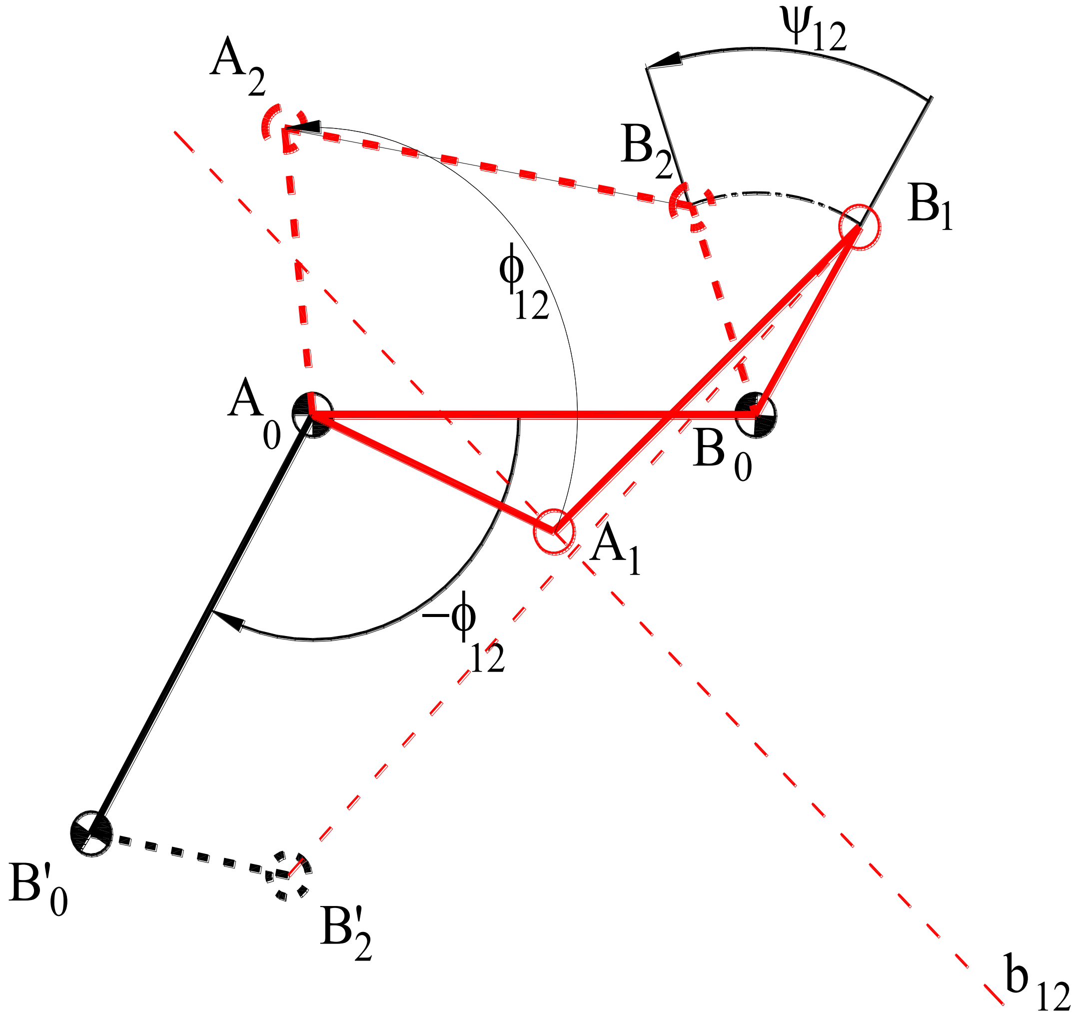


Kinematic Inversion





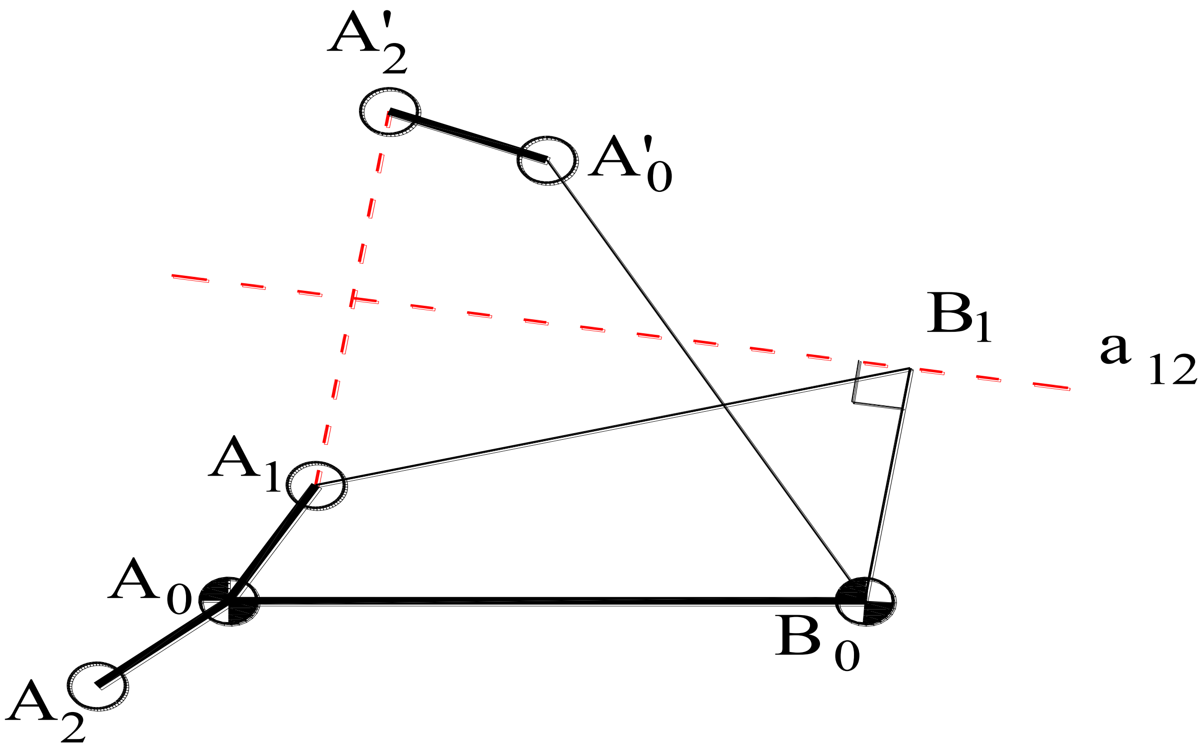
Relative motion of B_0B with respect to A_0A



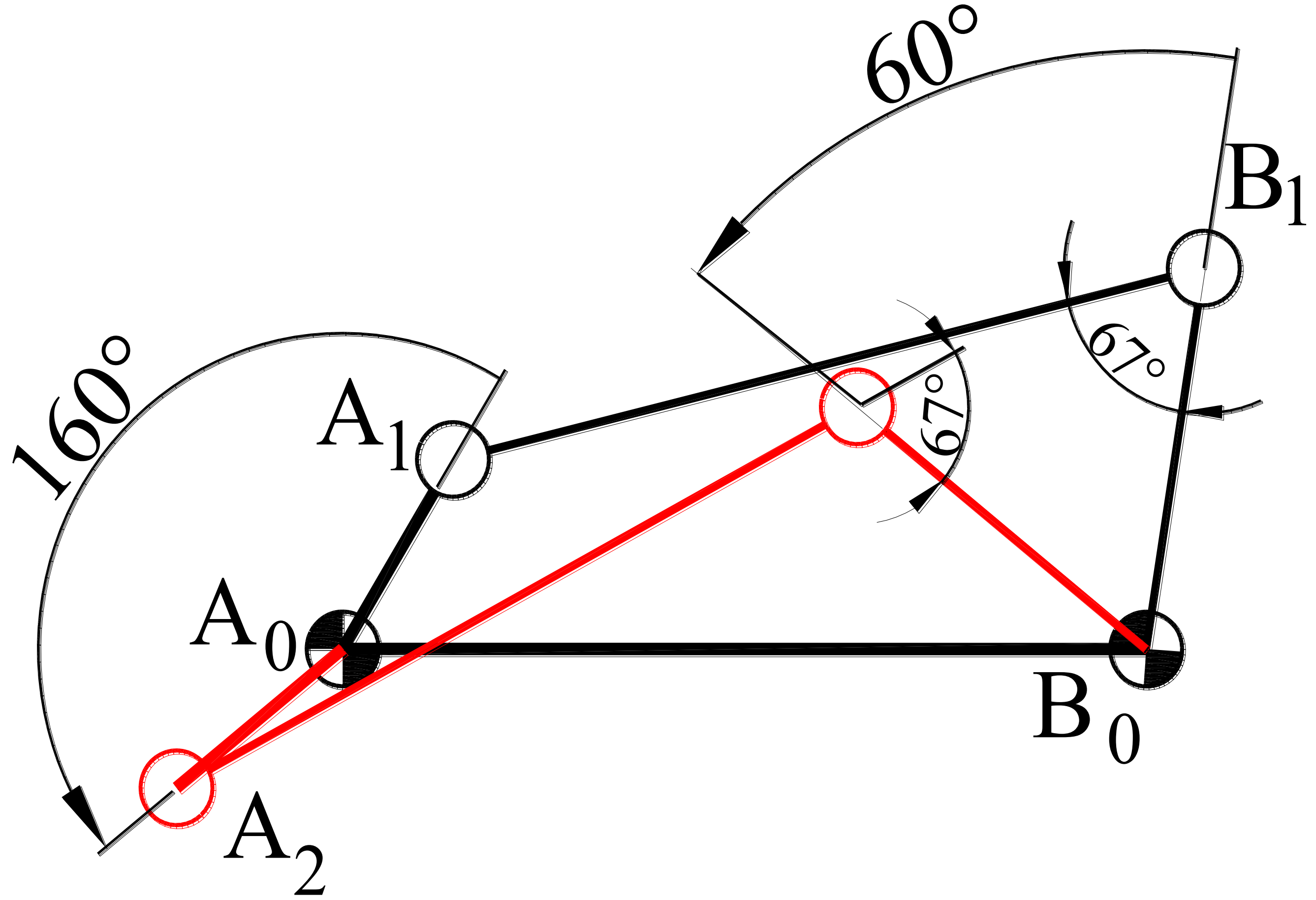
3. One can select A_1 anywhere on the perpendicular bisector to $B_1B'_2$. One possible solution is as shown. There is no guarantee for the motion in between the positions. For example, the above solution, although movable in between the two specifed positions will not be a good choice in an application (why?)

Example: When the input crank rotates by $\phi_{12}= 160^0$ (CCW), we want the output link to rotate by $\psi_{12}= 60^0$ (CCW).

1. Select A_0B_0 and A_1, determine A_2 such that $<A_1A_0A_2=\phi_{12}= 160^0$.
2. If we fix the output link B0B and look at the motion of A0A, in order to keep the relative positions the same, rotate $B_0A_0A_2$ about B_0 by -60^0 ($=-\psi_{12}$). Relative motion of A_0A with respect to B_0B_1 is the motion of A_0A from A_0A_1 to $A'_0A'_2$.
3. Draw the perpendicular bisector to $A_1A'_2$, a_{12} . B_1 can be selected anywhere on the perpendicular bisector a12.
4. İf B_0B_1 is selected such that it is perpendicular to a_{12} in the resulting mechanism will have 90^0 transmission angle in between the two positions and transmission angle will deviate less when moving in between the two positions.



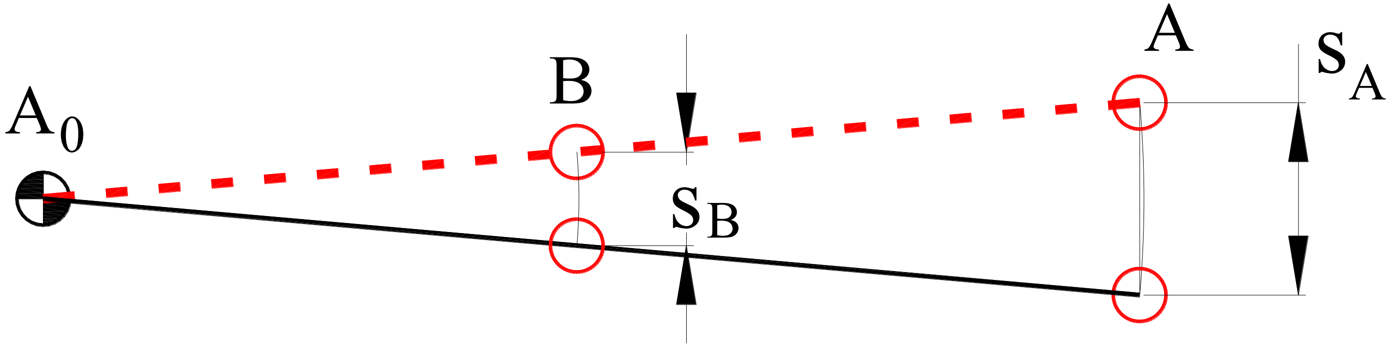
Result:



Geogebra

Design a four-bar mechanism such that when the input link rotates by 150^0 CW , output link will rotate by $60^{0.}$ CW.

Lever: (Archimedes (200 BC)

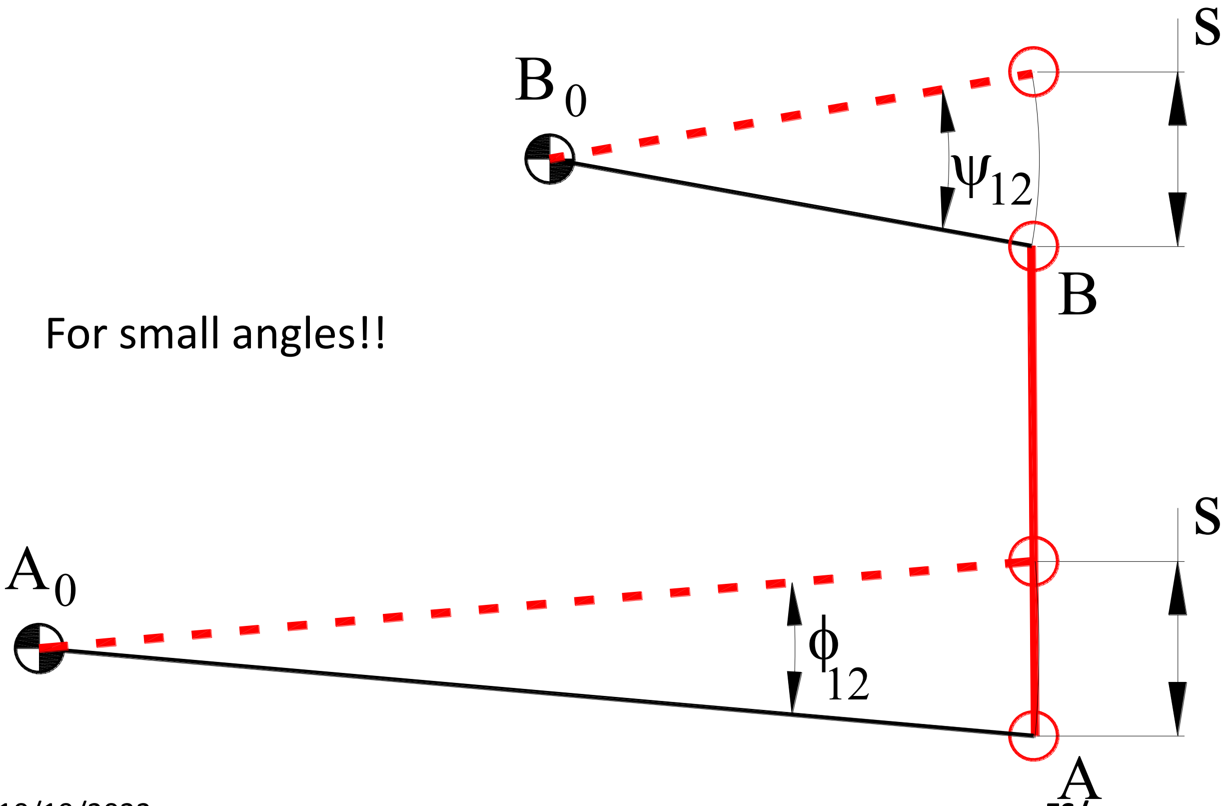


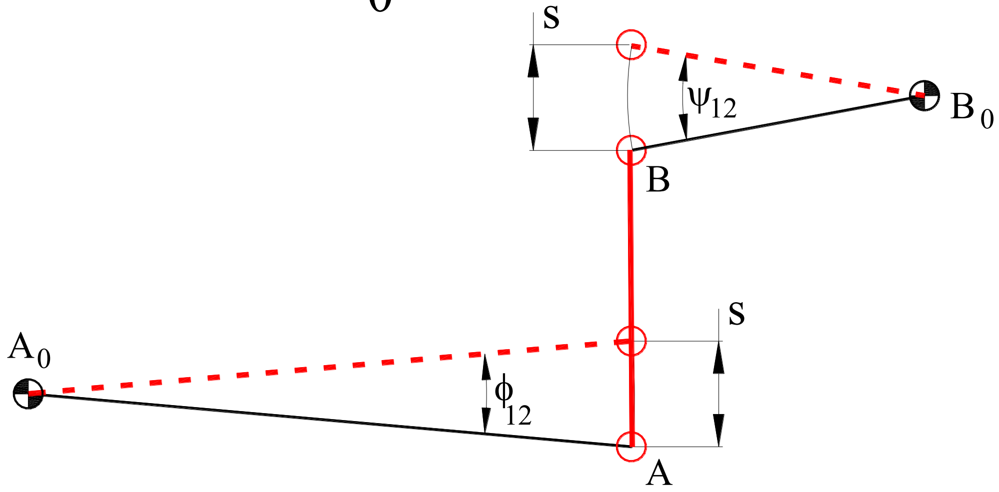
$$\frac{s_A}{s_B} = \frac{A_0A}{A_0B}$$

$$B_0B\psi_{12} \approx A_0A\phi_{12} \approx s$$

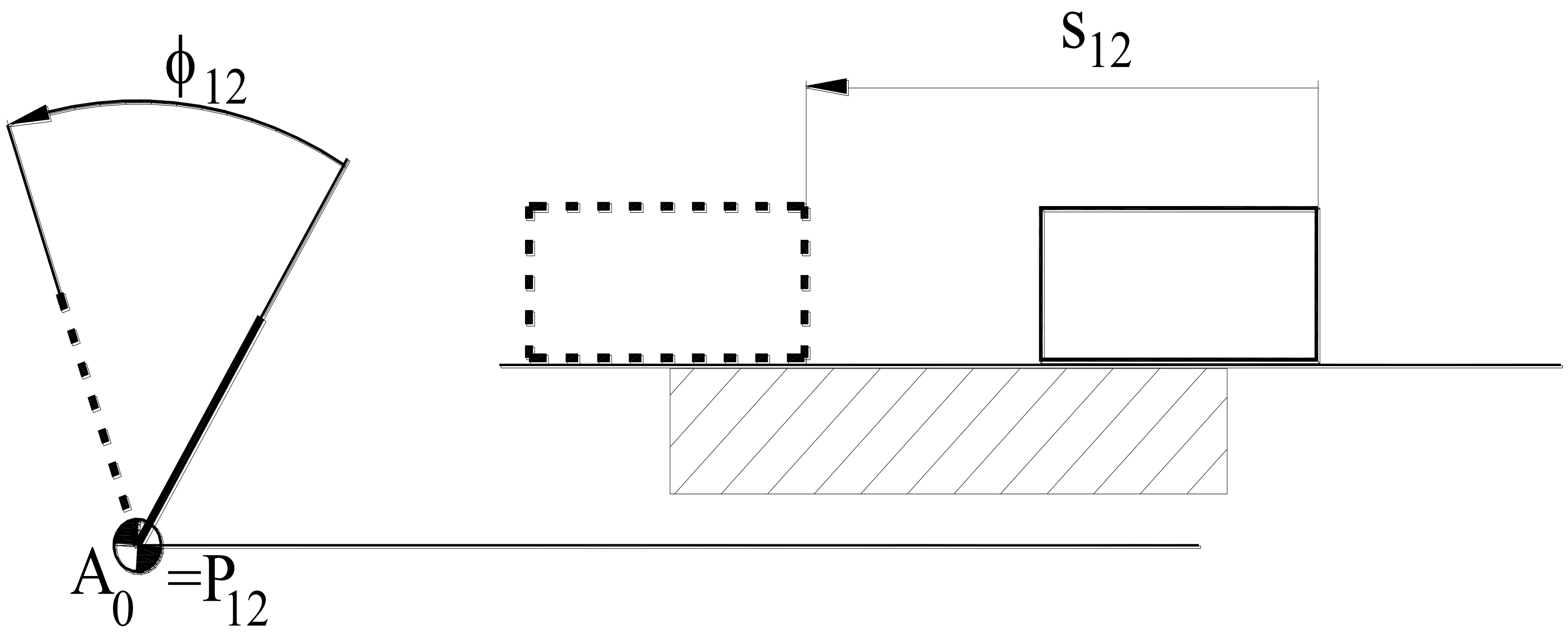
$$\psi_{12} \approx \frac{A_0A}{B_0B}\phi_{12}$$

For small angles!!





Correlation of slider displacement with crank angle

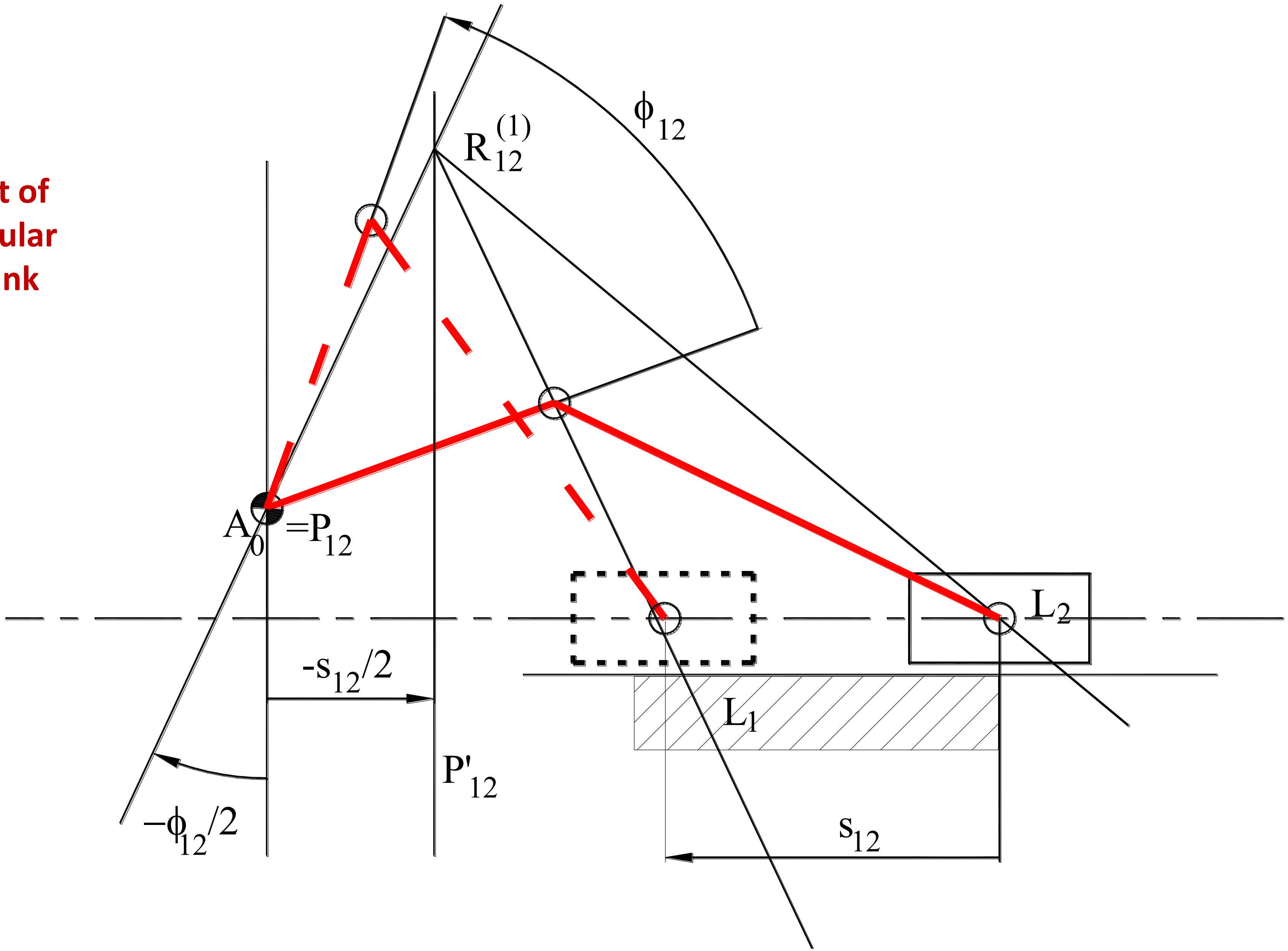


Given: the crank rotation and slider displacent for two positions: ϕ_{12} and s_{12}

Determine: a slider crank mechanism to realize this motion.

Method 1

Use linear displacement of the slide instead of angular displacement of the crank

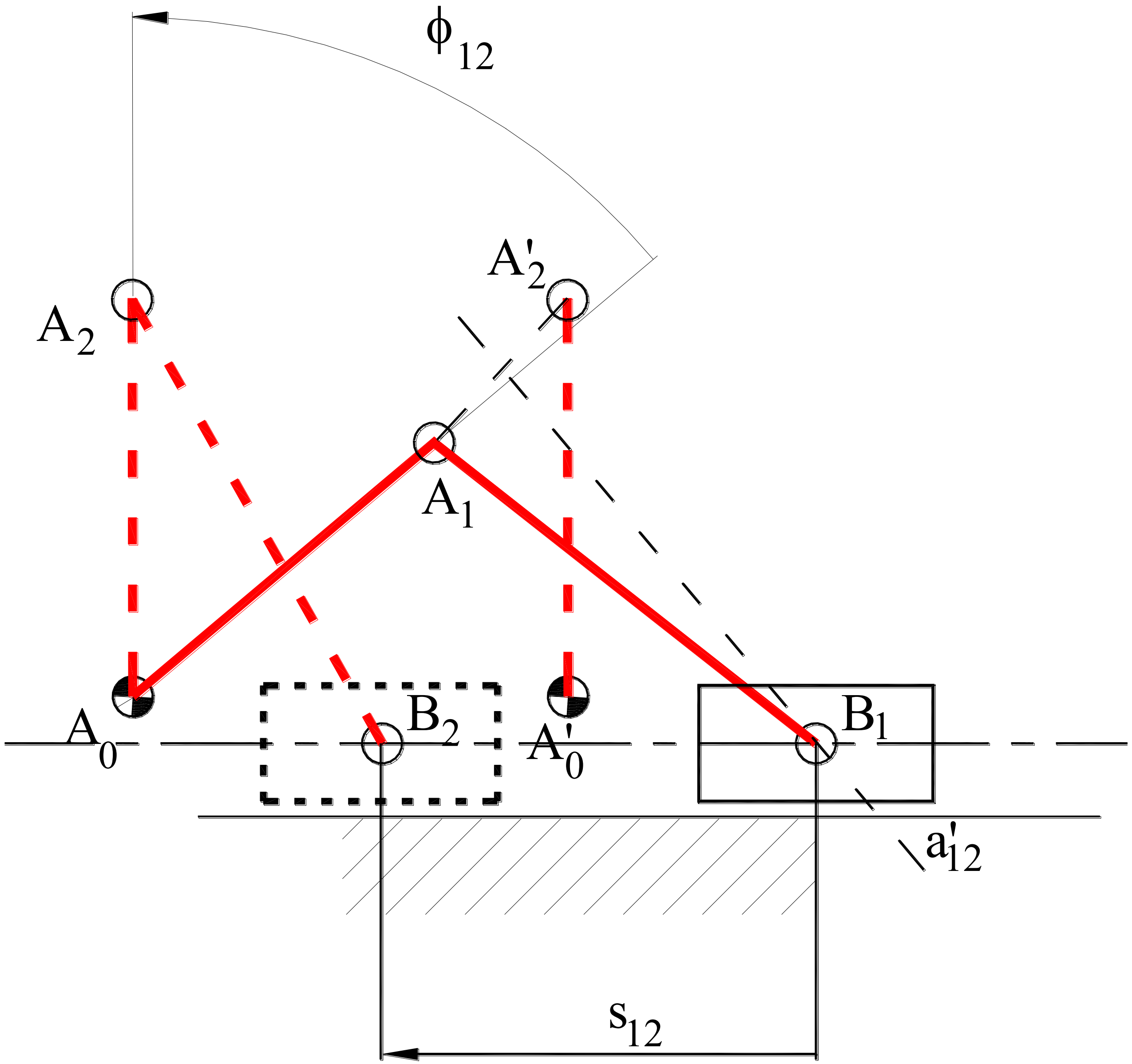


Method 2

1. Select A_0A_1 anywhere you like. Determine A_2 such that $<A_1A_0A_2=\phi_{12}$. A
2. Keep the crank fixed in first position. In order to have the same relative motion, move A_0A_2 by a distance $-s_{12}$. This will bring A_0A to $A'_0A'_2$. motion from A_0A_1 to $A'_0A'_2$ is the relative motion of plane A_0A w.r. to slider in the first position.
3. Select B_1 anywhere on the perpendicular bisector to $A_1A'_2$ (a'_{12}).

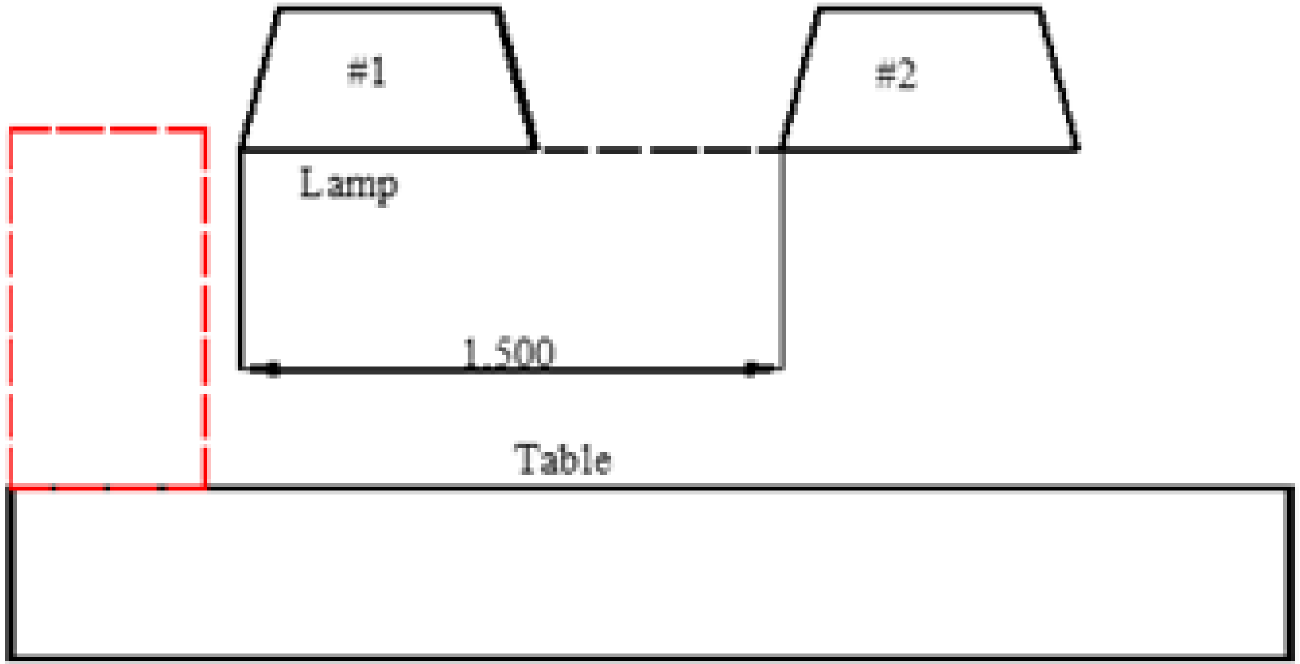
Where will you select B_1 in order to have a good transmission angle?

SonThe resulting mechanism must be checked!!!.



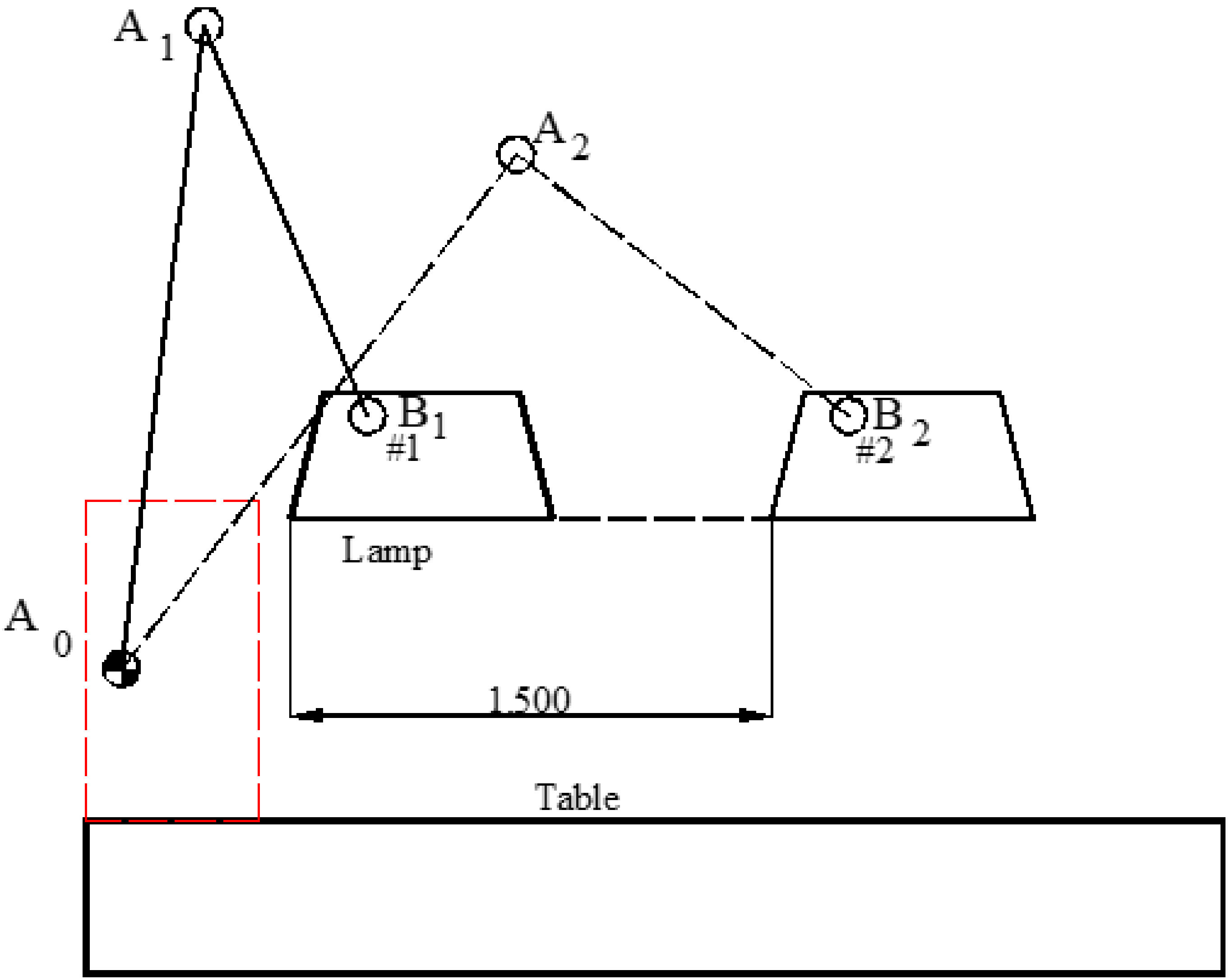
Design of Six Link Mechanisms

The fixed pivots must be within the rectangular area shown in red dashed line!!



Select the dyad A_0AB

This is a 3 dof open chain!

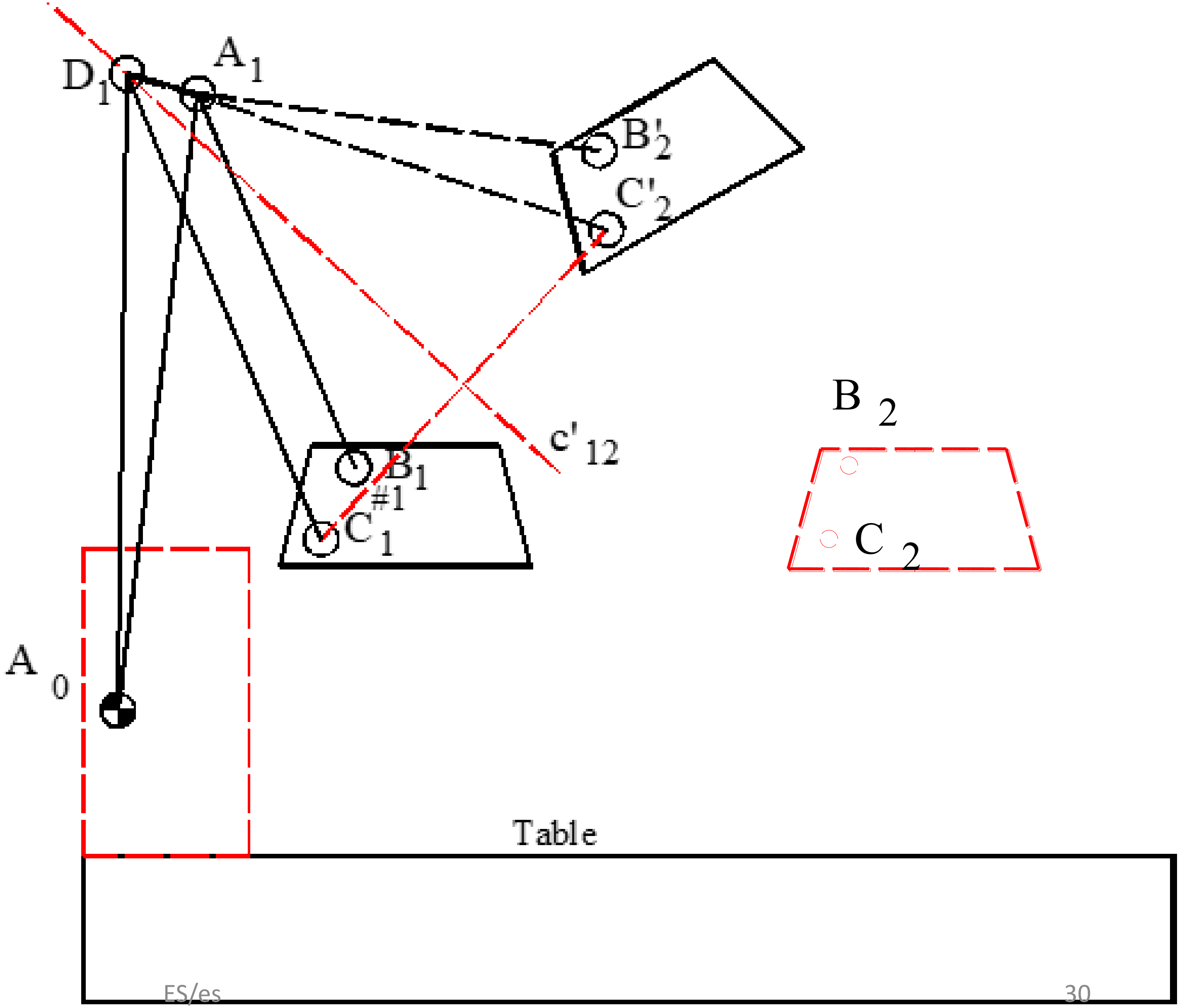


Relative Motion of the lamp with respect to A_0A

Select a point B_1 on the lamp. Determine B_2. ''SUBTRACT'' the motion of the crank A0A from the motion of the lamp (rotate the lamp about A_0 by an angle $<A_2A_0A_1$.). This will bring B to B'_2. Select the revolute joint axis A_1 anywhere on the perpendicular bisector to $B_1B'_2$.

If A_0A were fixed, (ABCD) will be a four-bar mechanism.

The system now has 2 dof!



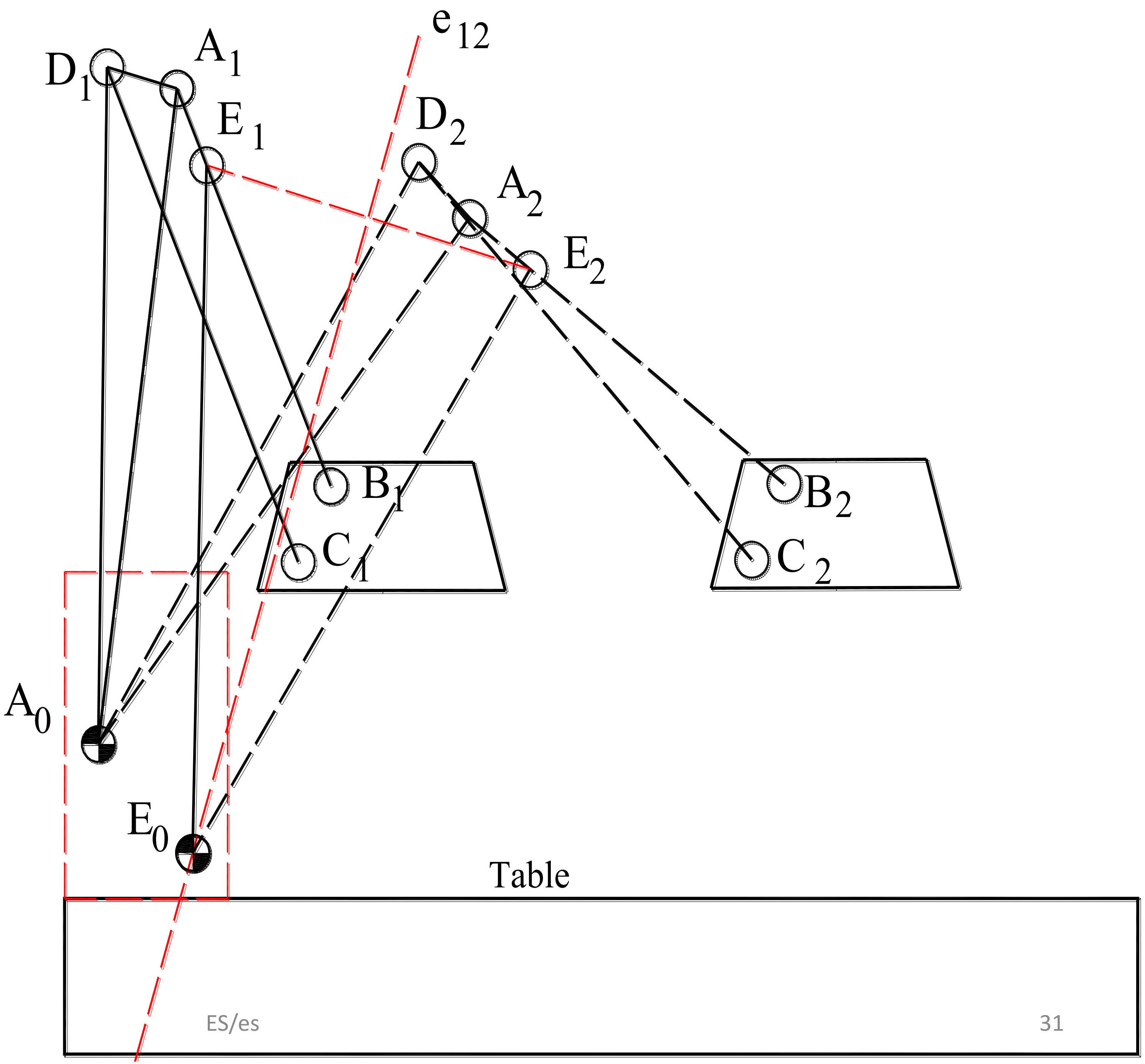
Next let us look at the motion of links AB or DC relative to the fixed plane.

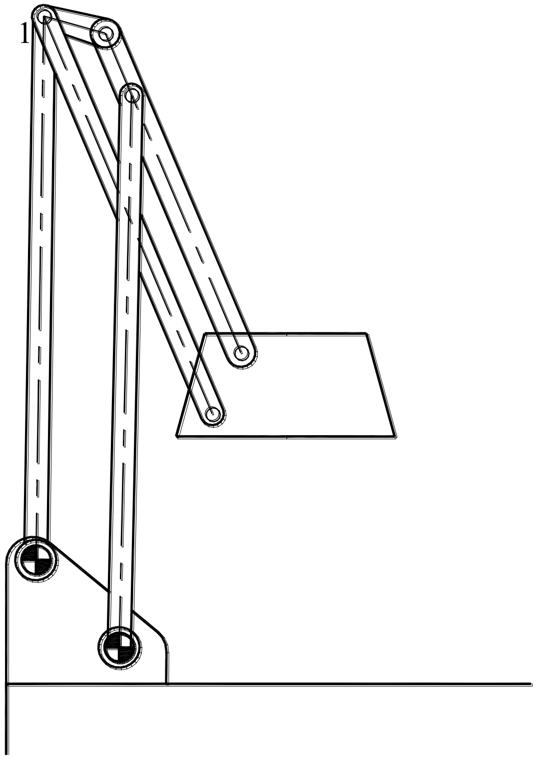
Select a point E on link AB (or DC). Determine the homologous (E_1 and E_2) .

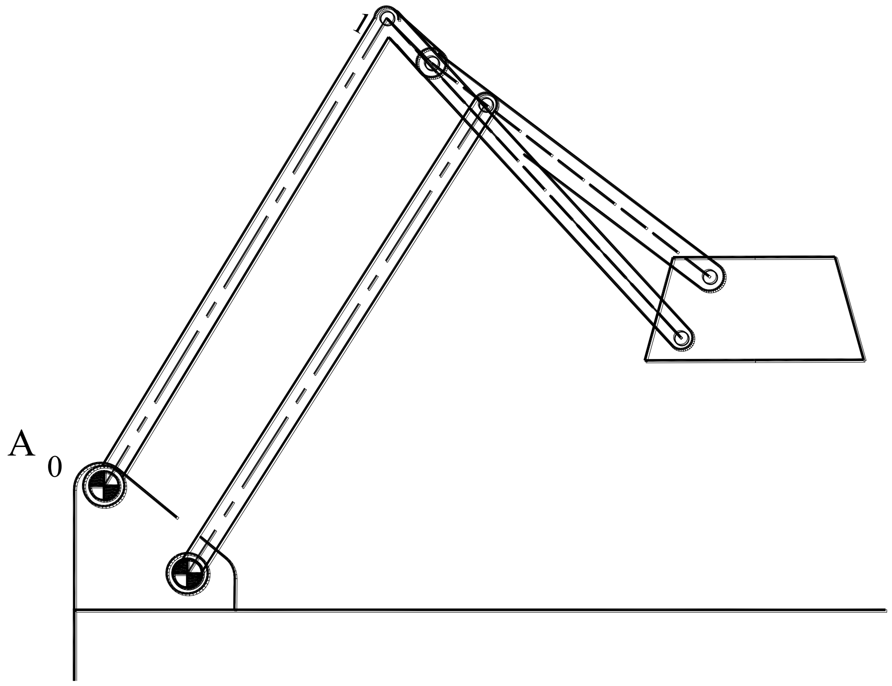
Select E_0 anywhere on the perpendicular bisector to E_1E_2 (e_{12}).

Now you can satisfy the condition «fixed pivots must be within the rectangular area shown»

Result must be checked!!.




1


1
A 0